6

Linear Inequations (In one variable)

Linear Inequations in One Variable

Inequalities of the form:

(i) $ax + b < c$ (ii) $ax + b \leq c$ (iii) $ax + b > c$ (iv) $ax + b \geq c$,

where a, b, c are real numbers, $a \neq 0$ and x is a variable, are called inequations in x.

Thus, each of the following is a linear inequation:

(i) $2x + 3 < 7$ (ii) $3x - 5 \leq 10$ (iii) $3x + 2 > 8$ (iv) $4x - 3 \geq 9$

Replacement Set or Domain of the Variable

A set given to us from which the values of *x* are replaced in an inequation in *x*, is called the *replacement set*.

SOLUTION SET The set of all those values of *x* taken from the replacement set which satisfy the given inequation is called the *solution set of the inequation*.

EXAMPLE *Write down the solution set of the inequation $x < 6$, when the replacement set is (i) N, (ii) W, (iii) Z.*

SOLUTION (i) Solution set = $\{x \in N : x < 6\} = \{1, 2, 3, 4, 5\}$.

(ii) Solution set = $\{x \in W : x < 6\} = \{0, 1, 2, 3, 4, 5\}$.

(iii) Solution set = $\{x \in Z : x < 6\} = \{5, 4, 3, 2, 1, 0, -1, -2, -3, \ldots\}$.

Rules for Solving an Inequation

RULE 1 Adding the same number or expression to each side of an inequation does not change the inequality.

RULE 2 Subtracting the same number or expression from each side of an inequation does not change the inequality.

RULE 3 Multiplying (or dividing) each side of an inequation by the same positive number does not change the inequality.

RULE 4 Multiplying (or dividing) each side of an inequation by the same negative number reverses the inequality.

Thus, $-x > 3 \Rightarrow x < -3$ [on multiplying both sides by –1].

And, $-3x \leq -12 \Rightarrow x \geq 4$ [on dividing both sides by –3].

TRANSPOSITION Using Rule 1 and Rule 2, we can drop any term from one side of an inequation and put it on the other side with the opposite sign.

This process is called *transposition*.

Thus, $x+3<2 \Rightarrow x<2-3 \Rightarrow x<-1$.

And, $2x-3\geq 6 \Rightarrow 2x\geq 6+3 \Rightarrow 2x\geq 9 \Rightarrow x\geq \frac{9}{2}$.

SOLVED EXAMPLES

EXAMPLE 1 *Solve* $5x<24$ *when (i)* $x\in N$, *(ii)* $x\in Z$.
In each case, represent the solution set on the number line.

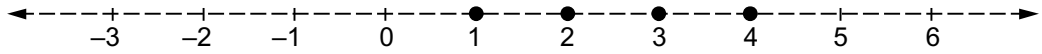
SOLUTION $5x<24 \Rightarrow x<\frac{24}{5}$

$\Rightarrow x<4.8$.

(i) Solution set $=\{x\in N: x<4.8\}$

$=\{1, 2, 3, 4\}$.

On the number line, we may represent it as shown below.



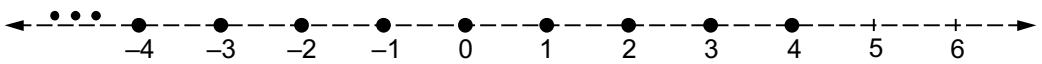
The darkened circles indicate the natural numbers contained in the set.

(ii) Solution set $=\{x\in Z: x<4.8\}$

$=\{4, 3, 2, 1, 0, -1, -2, -3, \ldots\}$

$=\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$.

On the number line, we may represent it as shown below.



The darkened circles show the integers contained in the set and three dark dots above the left part of the line show that the negative integers are continued idefinitely.

EXAMPLE 2 *Solve* $12+1\frac{5}{6}x\leq 5+3x$ *when (i)* $x\in N$, *(ii)* $x\in R$.
Draw the graph of the solution set in each case.

SOLUTION $12+1\frac{5}{6}x\leq 5+3x$

$\Rightarrow \quad 12+\frac{11}{6}x\leq 5+3x$

$\Rightarrow \quad 72+11x\leq 30+18x$ [multiplying both sides by 6]

$\Rightarrow \quad 11x\leq 18x-42$ [adding -72 to both sides]

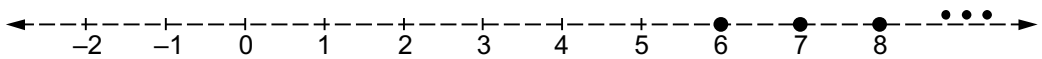
$\Rightarrow \quad -7x\leq -42$ [adding $-18x$ to both sides]

$\Rightarrow \quad x\geq 6$ [dividing both sides by -7].

(i) Solution set $=\{x\in N: x\geq 6\}$

$=\{6, 7, 8, 9, \ldots\}$.

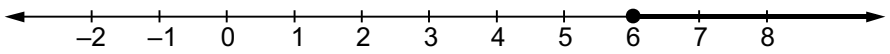
The graph of this set is the number line, shown below.



The darkened circles indicate the natural numbers contained in the set. Three dots above the right part of the line show that the natural numbers are continued indefinitely.

(ii) Solution set $= \{x \in R : x \geq 6\} = [6, \infty[$.

The graph of this set is shown below.



This graph consists of 6 and all real numbers greater than 6.

EXAMPLE 3 *Solve* $\frac{x+4}{x-2} > 0$ *and draw the graph of the solution set.*

SOLUTION Note that $\frac{a}{b} > 0$ when $(a > 0 \text{ and } b > 0)$ or $(a < 0 \text{ and } b < 0)$.

$\therefore$ either $(x+4 > 0 \text{ and } x-2 > 0)$ or $(x+4 < 0 \text{ and } x-2 < 0)$.

Case I *When* $x+4 > 0$ *and* $x-2 > 0$.

In this case, $x+4 > 0 \text{ and } x-2 > 0$

$\Rightarrow \quad x > -4 \text{ and } x > 2$

$\Rightarrow \quad x > 2$

$\Rightarrow \quad x \in (2, \infty)$.

Case II *When* $x+4 < 0$ *and* $x-2 < 0$.

In this case, $x+4 < 0 \text{ and } x-2 < 0$

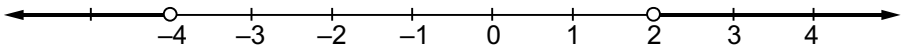
$\Rightarrow \quad x < -4 \text{ and } x < 2$

$\Rightarrow \quad x < -4$

$\Rightarrow \quad x \in (-\infty, -4)$.

$\therefore$ solution set $= (-\infty, -4) \cup (2, \infty)$.

The graph of the solution set is given below.



EXAMPLE 4 *Solve* $\frac{x-3}{x+4} < 0$ *and draw the graph of the solution set.*

SOLUTION Note that $\frac{a}{b} < 0$ when $(a > 0 \text{ and } b < 0)$ or $(a < 0 \text{ and } b > 0)$.

$\therefore$ either $(x-3 > 0 \text{ and } x+4 < 0)$ or $(x-3 < 0 \text{ and } x+4 > 0)$.

Case I *When* $x-3 > 0$ *and* $x+4 < 0$.

In this case, $x-3 > 0 \text{ and } x+4 < 0$

$\Rightarrow \quad x > 3 \text{ and } x < -4$.

This is not possible, as we can never find a real number which is greater than 3 and less than -4.

Case II *When* $x-3 < 0$ *and* $x+4 > 0$.

In this case, $x-3 < 0 \text{ and } x+4 > 0$

$\Rightarrow \quad x < 3 \text{ and } x > -4$

$\Rightarrow \quad -4 < x < 3 \Rightarrow x \in (-4, 3)$.

$\therefore$ solution set $= (-4, 3)$.

The graph of the solution set is given below.

EXAMPLE 5 *Solve* $\dfrac{x-2}{x+5} > 2.$

SOLUTION We have

$$\frac{x-2}{x+5} > 2$$

$$\Rightarrow \quad \frac{x-2}{x+5} - 2 > 0$$

$$\Rightarrow \quad \frac{x-2-2x-10}{x+5} > 0$$

$$\Rightarrow \quad \frac{-(x+12)}{(x+5)} > 0$$

$$\Rightarrow \quad \frac{x+12}{x+5} < 0 \qquad \text{[on multiplying both sides by } -1\text{]}.$$

$\therefore$ either $(x+12 < 0 \text{ and } x+5 > 0)$ or $(x+12 > 0 \text{ and } x+5 < 0)$.

Case I *When* $x+12 < 0$ *and* $x+5 > 0$.

In this case, $x+12 < 0$ and $x+5 > 0$

$\Rightarrow \quad x < -12$ and $x > -5$

$\Rightarrow \quad -5 < x < -12$, which is impossible $\qquad [\because -12 < -5]$.

Case II *When* $x+12 > 0$ *and* $x+5 < 0$.

In this case, $x+12 > 0$ and $x+5 < 0$

$\Rightarrow \quad x > -12$ and $x < -5$

$\Rightarrow \quad -12 < x < -5 \Rightarrow x \in (-12, -5)$.

$\therefore$ solution set $= (-12, -5)$.

EXAMPLE 6 *Solve* $\dfrac{5}{x-2} > 3$ *and represent the solution set on the number line.*

SOLUTION $\dfrac{5}{x-2} > 3 \Rightarrow \dfrac{5}{x-2} - 3 > 0$ [adding -3 to both sides]

$$\Rightarrow \frac{5-3x+6}{x-2} > 0$$

$$\Rightarrow \frac{11-3x}{x-2} > 0.$$

$\therefore$ either $(11-3x > 0 \text{ and } x-2 > 0)$ or $(11-3x < 0 \text{ and } x-2 < 0)$.

Case I *When* $11-3x > 0$ *and* $x-2 > 0$.

Now, $11-3x > 0$ and $x-2 > 0$

$\Rightarrow \quad -3x > -11$ and $x > 2$

$\Rightarrow \quad x < \dfrac{11}{3}$ and $x > 2$

$\Rightarrow \quad 2 < x < \dfrac{11}{3}.$ $\qquad \ldots$ (i)

Case II *When* $11-3x<0$ *and* $x-2<0$.

Now, $11-3x<0$ and $x-2<0$

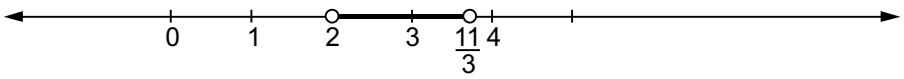
$\Rightarrow \quad -3x<-11$ and $x<2$

$\Rightarrow \quad x>\frac{11}{3}$ and $x<2$.

This is not possible, as we cannot find a real number which is greater than $\frac{11}{3}$ and less than 2.

$\therefore$ solution set $=\left\{x\in R: 2<x<\frac{11}{3}\right\}=\left(2,\frac{11}{3}\right)$.

We can represent this set on the number line, as given below.



SOME USEFUL RESULTS

Let a be a positive real number. Then,

(i) $|x|<a \Leftrightarrow -a<x<a \Leftrightarrow x\in(-a,a)$.

(ii) $|x|\le a \Leftrightarrow -a\le x\le a \Leftrightarrow x\in[-a,a]$.

(iii) $|x|>a \Leftrightarrow x<-a$ or $x>a$.

(iv) $|x|\ge a \Leftrightarrow x\le -a$ or $x\ge a$.

EXAMPLE 7 *Solve* $|4-x|<2$.

SOLUTION We have, $|x|<a \Leftrightarrow -a<x<a$.

$\therefore \quad |4-x|<2$

$\Leftrightarrow \quad -2<4-x<2$

$\Leftrightarrow \quad -2<4-x$ and $4-x<2$

$\Leftrightarrow \quad -2-4<-x$ and $-x<2-4$

$\Leftrightarrow \quad -6<-x$ and $-x<-2$

$\Leftrightarrow \quad 6>x$ and $x>2$

$\Leftrightarrow \quad 2<x<6$.

$\therefore$ solution set $=\{x\in R: 2<x<6\}=(2,6)$.

EXAMPLE 8 *Solve* $|3x-2|\le\frac{1}{2}, x\in R$.

SOLUTION We have, $|x|\le a \Leftrightarrow -a\le x\le a$.

$\therefore \quad |3x-2|\le\frac{1}{2} \Leftrightarrow -\frac{1}{2}\le 3x-2\le\frac{1}{2}$

$\Leftrightarrow \quad \frac{-1}{2}\le 3x-2$ and $3x-2\le\frac{1}{2}$

$\Leftrightarrow \quad \frac{-1}{2}+2\le 3x$ and $3x\le\frac{1}{2}+2$

$$\Leftrightarrow \frac{3}{2} \le 3x \text{ and } 3x \le \frac{5}{2}$$

$$\Leftrightarrow \frac{1}{2} \le x \text{ and } x \le \frac{5}{6}$$

$$\frac{1}{2} \le x \le \frac{5}{6}.$$

$\therefore$ solution set $= \left\{x \in R : \frac{1}{2} \le x \le \frac{5}{6}\right\} = \left[\frac{1}{2}, \frac{5}{6}\right].$

EXAMPLE 9 *Solve* $|x+1| > 4, x \in R.$

SOLUTION We have, $|x| > a \Leftrightarrow x < -a$ or $x > a.$

$\therefore$ $|x+1| > 4 \Leftrightarrow x+1 < -4$ or $x+1 > 4$

$\Leftrightarrow x < -4-1$ or $x > 4-1$

$\Leftrightarrow x < -5$ or $x > 3$

$\Leftrightarrow x \in (-\infty, -5)$ or $x \in (3, \infty).$

$\therefore$ solution set $= (-\infty, -5) \cup (3, \infty).$

EXAMPLE 10 *Solve* $|3-4x| \ge 9, x \in R.$

SOLUTION We have, $|x| \ge a \Leftrightarrow x \le -a$ or $x \ge a.$

$\therefore$ $|3-4x| \ge 9 \Leftrightarrow 3-4x \le -9$ or $3-4x \ge 9$

$\Leftrightarrow -4x \le -9-3$ or $-4x \ge 9-3$

$\Leftrightarrow -4x \le -12$ or $-4x \ge 6$

$\Leftrightarrow x \ge 3$ or $x \le \frac{-3}{2}$

$\Leftrightarrow x \le \frac{-3}{2}$ or $x \ge 3$

$\Leftrightarrow x \in \left(-\infty, \frac{-3}{2}\right]$ or $x \in [3, \infty).$

$\therefore$ solution set $= \left(-\infty, \frac{-3}{2}\right] \cup [3, \infty).$

EXAMPLE 11 *Solve* $\frac{2}{|x-3|} > 5, x \in R.$

SOLUTION Clearly, $x-3 \neq 0$ and therefore, $x \neq 3.$

We have, $\frac{2}{|x-3|} > 5.$ … (i)

Since $|x-3|$ is positive, we may multiply both sides of (i) by $|x-3|$. This gives

$2 > 5\ |x-3|$

$\Leftrightarrow \frac{2}{5} > |x-3|$

$\Leftrightarrow |x-3| < \frac{2}{5}$

$\Leftrightarrow \frac{-2}{5} < x-3 < \frac{2}{5}$ $[\because |x| < a \Leftrightarrow -a < x < a]$

$$\Leftrightarrow \quad \frac{-2}{5} < x-3 \text{ and } x-3 < \frac{2}{5}$$

$$\Leftrightarrow \quad \frac{-2}{5}+3 < x \text{ and } x < \frac{2}{5}+3$$

$$\Leftrightarrow \quad \frac{13}{5} < x \text{ and } x < \frac{17}{5}.$$

$$\Leftrightarrow \quad \frac{13}{5} < x < \frac{17}{5}.$$

Also, as shown above, $x \neq 3$.

$$\therefore \quad \text{solution set} = \left\{x \in R : \frac{13}{5} < x < \frac{17}{5}\right\} - \{3\}$$

$$= (2.6, 3.4) - \{3\} = (2.6, 3) \cup (3, 3.4).$$

EXAMPLE 12 *Solve* $\dfrac{|x-2|-1}{|x-2|-2} \leq 0, x \in R.$

SOLUTION We have either $x-2 \geq 0$ or $x-2 < 0$.

Case I *When* $x-2 \geq 0$.

In this case, $x \geq 2$ and $|x-2| = x-2$.

So, the given inequation becomes

$$\frac{x-2-1}{x-2-2} \leq 0 \Rightarrow \frac{x-3}{x-4} \leq 0.$$

$\therefore \quad (x-3 \geq 0 \text{ and } x-4 < 0) \text{ or } (x-3 \leq 0 \text{ and } x-4 > 0)$

$\Rightarrow \quad (x \geq 3 \text{ and } x < 4) \text{ or } (x < 3 \text{ and } x > 4)$

$\Rightarrow \quad 3 \leq x < 4 \qquad [\because\ x < 3 \text{ and } x > 4 \text{ is not possible}]$

$\Rightarrow \quad x \in [3, 4) \qquad [\text{this includes } x \geq 2].$

Case II *When* $x-2 < 0$.

In this case, $x < 2$ and $|x-2| = -(x-2) = -x+2$.

So, the given inequation becomes

$$\frac{-x+2-1}{-x+2-2} \leq 0 \Rightarrow \frac{-x+1}{-x} \leq 0$$

$$\Rightarrow \frac{x-1}{x} \leq 0$$

[multiplying num. and denom. by –1]

$\therefore \quad (x-1 \leq 0 \text{ and } x > 0) \text{ or } (x-1 \geq 0 \text{ and } x < 0)$

$\Rightarrow \quad (x \leq 1 \text{ and } x > 0) \text{ or } (x \geq 1 \text{ and } x < 0)$

$\Rightarrow \quad 0 < x \leq 1 \qquad [\because\ x \geq 1 \text{ and } x < 0 \text{ is not possible}]$

$\Rightarrow \quad x \in (0, 1] \qquad [\text{this includes } x < 2].$

Hence, from the above two cases, we get

solution set $= (0, 1] \cup [3, 4)$.

EXAMPLE 13 *Solve* $\dfrac{|x+3|+x}{(x+2)} > 1, x \in R.$

SOLUTION We have

$$\frac{|x+3|+x}{(x+2)} > 1$$

$$\Rightarrow \quad \frac{|x+3|+x}{(x+2)} - 1 > 0$$

$$\Rightarrow \quad \frac{|x+3|+x-x-2}{(x+2)} > 0$$

$$\Rightarrow \quad \frac{|x+3|-2}{(x+2)} > 0. \qquad \ldots \text{(i)}$$

Now, we have either $x+3 \geq 0$ or $x+3 < 0$.

Case I *When* $x+3 \geq 0$.

In this case, $x \geq -3$ and $|x+3| = x+3$.

$\therefore$ (i) becomes

$$\frac{x+3-2}{x+2} > 0 \Rightarrow \frac{x+1}{x+2} > 0.$$

$\therefore \quad (x+1 > 0 \text{ and } x+2 > 0) \text{ or } (x+1 < 0 \text{ and } x+2 < 0)$

$\Rightarrow \quad (x > -1 \text{ and } x > -2) \text{ or } (x < -1 \text{ and } x < -2)$

$\Rightarrow \quad (x > -1) \text{ or } (x < -2)$

$\Rightarrow \quad (x < -2) \text{ or } (x > -1)$

$\Rightarrow \quad x \in (-\infty, -2) \text{ or } x \in (-1, \infty)$

$\Rightarrow \quad x \in [-3, -2) \text{ or } x \in (-1, \infty) \qquad [\because\ x \geq -3]$

$\Rightarrow \quad x \in [-3, -2) \cup (-1, \infty).$

Case II *When* $x+3 < 0$.

In this case, $x < -3$ and $|x+3| = -(x+3) = -x-3$.

$\therefore$ (i) becomes

$$\frac{-x-3-2}{x+2} > 0 \Rightarrow \frac{-(x+5)}{(x+2)} > 0 \Rightarrow \frac{x+5}{x+2} < 0.$$

$\therefore \quad (x+5 < 0 \text{ and } x+2 > 0) \text{ or } (x+5 > 0 \text{ and } x+2 < 0)$

$\Rightarrow \quad (x < -5 \text{ and } x > -2) \text{ or } (x > -5 \text{ and } x < -2)$

$\Rightarrow \quad -5 < x < -2 \qquad [\because\ x < -5 \text{ and } x > -2 \text{ is not possible}]$

$\Rightarrow \quad -5 < x < -3 \qquad [\because\ x < -3]$

$\Rightarrow \quad x \in (-5, -3).$

$\therefore$ solution set $= (-5, -3) \cup [-3, -2) \cup (-1, \infty)$

$= (-5, -2) \cup (-1, \infty).$

EXAMPLE 14 *Solve* $|x-1|+|x-2| \geq 4, x \in R.$

SOLUTION Putting $x-1=0$ and $x-2=0$, we get $x=1$ and $x=2$ as the critical points. These points divide the whole real line into three parts, namely $(-\infty, 1), [1, 2)$ and $[2, \infty)$. So, we consider the following three cases.

Case I *When* $-\infty < x < 1$.

In this case, $x-1<0$ and $x-2<0$.

$\therefore \quad |x-1| = -(x-1) = -x+1$ and $|x-2| = -(x-2) = -x+2$.

Now, $|x-1|+|x-2| \geq 4$

$\Rightarrow \quad -x+1-x+2 \geq 4$

$\Rightarrow \quad -2x+3 \geq 4 \Rightarrow -2x \geq 4-3 \Rightarrow -2x \geq 1 \Rightarrow x \leq \frac{-1}{2}$

$\Rightarrow \quad x \in \left(-\infty, \frac{-1}{2}\right]$.

But, $-\infty < x < 1$.

$\therefore$ solution set in this case $= \left(-\infty, \frac{-1}{2}\right] \cap (-\infty, 1) = \left(-\infty, \frac{-1}{2}\right]$.

Case II *When* $1 \leq x < 2$.

In this case, $x-1 \geq 0$ and $x-2<0$.

$\therefore \quad |x-1| = x-1$ and $|x-2| = -(x-2) = -x+2$.

Now, $|x-1|+|x-2| \geq 4$

$\Rightarrow \quad x-1-x+2 \geq 4 \Rightarrow -1 \geq 4$, which is absurd.

So, the given inequation has no solution in [1, 2).

Case III *When* $2 \leq x < \infty$.

In this case, $x-2 \geq 0$ and $x-1>0$.

$\therefore \quad |x-2| = x-2$ and $|x-1| = x-1$.

Now, $|x-1|+|x-2| \geq 4$

$\Rightarrow \quad x-1+x-2 \geq 4 \Rightarrow 2x-3 \geq 4 \Rightarrow 2x \geq 7 \Rightarrow x \geq \frac{7}{2}$.

Also, in this case, we have $x \geq 2$.

$\therefore$ solution set in this case $= \left[\frac{7}{2}, \infty\right) \cap [2, \infty) = \left[\frac{7}{2}, \infty\right)$.

Hence, from all the above cases, we have

solution set $= \left(-\infty, \frac{-1}{2}\right] \cup \left[\frac{7}{2}, \infty\right)$.

SOLVING SIMULTANEOUS INEQUATIONS IN ONE VARIABLE

METHOD Suppose we have to solve two inequations simultaneously. Find the solution set of each of them. Then, the intersection of these solution sets is the required solution set.

EXAMPLE 15 *Solve the inequations* $-3 \leq 3-2x < 9, x \in R$. *Represent the solution set on the real line.*

SOLUTION We have

$$-3 \leq 3-2x < 9 \Rightarrow -3 \leq 3-2x \text{ and } 3-2x < 9.$$

Now, $-3 \leq 3-2x \Rightarrow -3+2x \leq 3$ [adding $2x$ on both sides]

$\Rightarrow 2x \leq 6$ [adding 3 on both sides]

$\Rightarrow x \leq 3$ [dividing both sides by 2]

$\Rightarrow x \in (-\infty, 3]$.

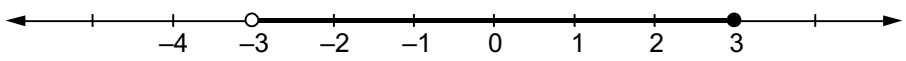
And, $3-2x<9 \Rightarrow -2x<6$ [subtracting 3 from both sides]

$\Rightarrow x>-3$ [dividing both sides by -2]

$\Rightarrow x\in(-3,\infty)$.

$\therefore$ solution set $=(-\infty,3]\cap(-3,\infty)=(-3,3]$.

We may represent it on the number line as shown below.



EXAMPLE 16 *Solve the inequations* $2x-3<x+2\le 3x+5, x\in R$. *Draw the graph of the solution set.*

SOLUTION We have

$$2x-3<x+2\le 3x+5 \Rightarrow 2x-3<x+2 \text{ and } x+2\le 3x+5.$$

Now, $2x-3<x+2 \Rightarrow x-3<2$ [adding $-x$ on both sides]

$\Rightarrow x<5$ [adding 3 on both sides]

$\Rightarrow x\in(-\infty,5)$.

And, $x+2\le 3x+5 \Rightarrow 3x+5\ge x+2$ [$\because a\le b \Rightarrow b\ge a$]

$\Rightarrow 2x+5\ge 2$ [adding $-x$ on both sides]

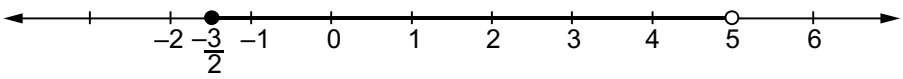
$\Rightarrow 2x\ge -3$ [adding -5 on both sides]

$\Rightarrow x\ge \frac{-3}{2}$ [dividing both sides by 2]

$\Rightarrow x\in\left[\frac{-3}{2},\infty\right)$.

$\therefore$ solution set $=(-\infty,5)\cap\left[\frac{-3}{2},\infty\right)=\left[\frac{-3}{2},5\right)$.

We may represent it on the number line, as shown below.



EXAMPLE 17 *Solve the system of inequations* $2x-1>x+\frac{7-x}{3}>2, x\in R$.

Represent the solution set on the number line.

SOLUTION We have

$$2x-1>x+\frac{7-x}{3}>2 \Rightarrow 2x-1>x+\frac{7-x}{3} \text{ and } x+\frac{7x}{3}>2.$$

Now, $2x-1>x+\frac{7-x}{3} \Rightarrow 6x-3>3x+7-x$

[multiplying both sides by 3]

$\Rightarrow 6x-3>2x+7$

$\Rightarrow 4x-3>7$ [adding $-2x$ on both sides]

$\Rightarrow 4x>10$ [adding 3 on both sides]

$\Rightarrow x>\frac{5}{2}$ [dividing both sides by 4]

$\Rightarrow x\in\left(\frac{5}{2},\infty\right)$.

And, $x+\frac{7-x}{3}>2 \Rightarrow 3x+7-x>6$ [multiplying both sides by 3]

$\Rightarrow 2x+7>6$

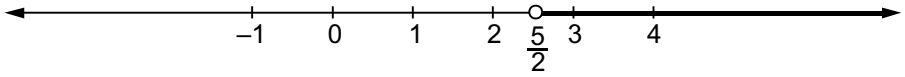
$\Rightarrow 2x>-1$ [adding –7 on both sides]

$\Rightarrow x>\frac{-1}{2}$ [dividing both sides by 2]

$\Rightarrow x\in\left(\frac{-1}{2},\infty\right)$.

$\therefore$ solution set $=\left(\frac{5}{2},\infty\right)\cap\left(\frac{-1}{2},\infty\right)=\left(\frac{5}{2},\infty\right)$.

The solution set on the number line may be represented as shown below.



EXAMPLE 18 *Solve the system of inequations* $-5\le\frac{2-3x}{4}\le 9$.

SOLUTION We have

$$-5\le\frac{2-3x}{4}\le 9 \Rightarrow -5\le\frac{2-3x}{4} \text{ and } \frac{2-3x}{4}\le 9.$$

Now, $-5\le\frac{2-3x}{4} \Rightarrow -20\le 2-3x$ [multiplying both sides by 4]

$\Rightarrow -22\le -3x$ [adding -2 on both sides]

$\Rightarrow 22\ge 3x$ [multiplying both sides by -1]

$\Rightarrow 3x\le 22$

$\Rightarrow x\le\frac{22}{3}$ [dividing both sides by 3]

$\Rightarrow x\in\left(-\infty,\frac{22}{3}\right]$.

And, $\frac{2-3x}{4}\le 9 \Rightarrow 2-3x\le 36$ [multiplying both sides by 4]

$\Rightarrow -3x\le 34$

$\Rightarrow x\ge\frac{-34}{3}$ [dividing both sides by -3]

$\Rightarrow x\in\left[\frac{-34}{3},\infty\right)$.

$\therefore$ solution set $=\left(-\infty,\frac{22}{3}\right]\cap\left[\frac{-34}{3},\infty\right)=\left[\frac{-34}{3},\frac{22}{3}\right]$.

EXAMPLE 19 *Solve the system of inequations:*

$$\frac{6x}{4x-1}<\frac{1}{2},\ \frac{x}{2x+1}\ge\frac{1}{4}.$$

SOLUTION The first inequation of the given system is

$$\frac{6x}{4x-1}<\frac{1}{2}$$

$$\Rightarrow \quad \frac{6x}{4x-1}-\frac{1}{2}<0$$

$$\Rightarrow \quad \frac{12x-4x+1}{2(4x-1)}<0$$

$$\Rightarrow \quad \frac{8x+1}{2(4x-1)}<0 \Rightarrow \frac{8x+1}{4x-1}<0. \qquad \ldots \text{(i)}$$

$$\therefore \quad (8x+1>0 \text{ and } 4x-1<0) \text{ or } (8x+1<0 \text{ and } 4x-1>0)$$

$$\Rightarrow \quad (8x>-1 \text{ and } 4x<1) \text{ or } (8x<-1 \text{ and } 4x>1)$$

$$\Rightarrow \quad \left(x>\frac{-1}{8} \text{ and } x<\frac{1}{4}\right) \text{ or } \left(x<\frac{-1}{8} \text{ and } x>\frac{1}{4}\right)$$

$$\Rightarrow \quad \frac{-1}{8}<x<\frac{1}{4} \qquad \left[\because\ x<\frac{-1}{8} \text{ and } x>\frac{1}{4} \text{ is not possible}\right]$$

$$\Rightarrow \quad x\in\left(\frac{-1}{8},\frac{1}{4}\right).$$

The second inequation of the given system is

$$\frac{x}{2x+1}\geq\frac{1}{4}$$

$$\Rightarrow \quad \frac{x}{2x+1}-\frac{1}{4}\geq 0$$

$$\Rightarrow \quad \frac{4x-2x-1}{4(2x+1)}\geq 0 \Rightarrow \frac{2x-1}{2x+1}\geq 0. \qquad \ldots \text{(ii)}$$

$$\therefore \quad (2x-1\leq 0 \text{ and } 2x+1<0) \text{ or } (2x-1\geq 0 \text{ and } 2x+1>0)$$

$$\Rightarrow \quad (2x\leq 1 \text{ and } 2x<-1) \text{ or } (2x\geq 1 \text{ and } 2x>-1)$$

$$\Rightarrow \quad \left(x\leq\frac{1}{2} \text{ and } x<\frac{-1}{2}\right) \text{ or } \left(x\geq\frac{1}{2} \text{ and } x>\frac{-1}{2}\right)$$

$$\Rightarrow \quad \left(x<\frac{-1}{2}\right) \text{ or } \left(x\geq\frac{1}{2}\right)$$

$$\Rightarrow \quad x\in\left(-\infty,\frac{-1}{2}\right) \text{ or } x\in\left[\frac{1}{2},\infty\right)$$

$$\Rightarrow \quad x\in\left(-\infty,\frac{-1}{2}\right)\cup\left[\frac{1}{2},\infty\right)$$

$$\therefore \quad \text{solution set}=\left(\frac{-1}{8},\frac{1}{4}\right)\cap\left\{\left(-\infty,\frac{-1}{2}\right)\cup\left[\frac{1}{2},\infty\right)\right\}=\phi.$$

Hence, the given system of inequations has no solution.

EXAMPLE 20 *Solve the system of inequations:* $|x-1|\leq 5,\ |x|\geq 2.$

SOLUTION The first inequation is $|x-1|\leq 5.$

Using $|x|\leq a \Leftrightarrow -a\leq x\leq a$, we get

$$|x-1|\leq 5 \Rightarrow -5\leq x-1\leq 5$$

$$\Rightarrow -4\leq x\leq 6 \Rightarrow x\in[-4,6].$$

The second inequation is $|x|\geq 2.$

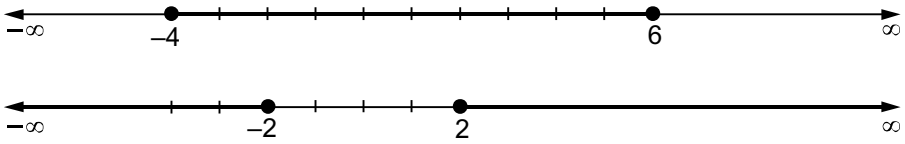
Using $|x|\geq a \Leftrightarrow x\leq -a \text{ or } x\geq a$, we get

$|x| \geq 2 \Rightarrow x \leq -2$ or $x \geq 2$

$\Rightarrow x \in (-\infty, -2] \cup [2, \infty)$.

$\therefore$ solution set for the given system is

$\{(-\infty, -2] \cup [2, \infty)\} \cap [-4, 6] = [-4, -2] \cup [2, 6]$.



WORD PROBLEMS

EXAMPLE 1 *Find all pairs of consecutive odd positive integers both of which are smaller than* 10 *such that their sum is more than* 11.

SOLUTION Let the required consecutive odd positive integers be x and $x+2$. Then,

$x + 2 < 10$ and $x + (x+2) > 11$

$\Rightarrow$ $x < 8$ and $2x + 2 > 11$

$\Rightarrow$ $x < 8$ and $2x > 9$

$\Rightarrow$ $x < 8$ and $x > \frac{9}{2}$

$\Rightarrow$ $\frac{9}{2} < x < 8 \Rightarrow 4.5 < x < 8$.

$\therefore$ x can take the odd integral values 5 and 7.

Hence, the required pairs of odd integers are (5, 7) and (7, 9).

EXAMPLE 2 *Find all pairs of consecutive even positive integers both of which are larger than* 5 *such that their sum is less than* 23.

SOLUTION Let the required consecutive even positive integers be x and $x+2$. Then,

$x > 5$ and $x + (x+2) < 23$

$\Rightarrow$ $x > 5$ and $2x < 21$

$\Rightarrow$ $5 < x$ and $x < 10.5$

$\Rightarrow$ $5 < x < 10.5$.

$\therefore$ x can take the even integral values 6, 8 and 10.

Hence, the required pairs of even integers are (6, 8), (8, 10) and (10, 12).

EXAMPLE 3 *The longest side of a triangle is* 3 *times the shortest side and the third side is* 2 cm *shorter than the longest side. If the perimeter of the triangle is at least* 61 cm, *find the minimum length of the shortest side.*

SOLUTION Let the minimum length of the shortest side be x cm.

Then, the longest side = $3x$ cm and the third side = $(3x-2)$ cm.

$\therefore$ $x + 3x + (3x-2) \geq 61 \Rightarrow 7x \geq 63 \Rightarrow x \geq 9$.

Hence, the minimum length of the shortest side is 9 cm.

EXAMPLE 4 *The cost and revenue functions of a product are given by* $C(x) = 20x + 4000$ *and* $R(x) = 60x + 2000$ *respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?*

SOLUTION Let the required number of items sold be *x* and let $P(x)$ be the profit function. Then,

profit function = (revenue function) – (cost function)

$\Rightarrow \quad P(x) = (60x + 2000) - (20x + 4000)$

$\Rightarrow \quad P(x) = 40x - 2000.$

For some profit, we have $P(x) > 0$.

$\therefore \quad 40x - 2000 > 0$

$\Rightarrow \quad 40x > 2000$

$\Rightarrow \quad x > 50.$

Hence, the manufacturer must sell more than 50 items to realise some profit.

EXAMPLE 5 *A man wants to cut three lengths from a single piece of cloth of length* 91 cm. *The second length is to be* 3 cm *longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest piece of cloth if the third piece is to be at least* 5 cm *longer than the second.*

SOLUTION Let the length of the shortest piece be *x* cm. Then, second length = $(x + 3)$ cm and third length = $2x$ cm.

$\therefore \quad x + (x + 3) + 2x \leq 91$ and $2x \geq (x + 3) + 5$

$\Rightarrow \quad 4x + 3 \leq 91$ and $2x \geq x + 8$

$\Rightarrow \quad 4x \leq 91 - 3$ and $2x - x \geq 8$

$\Rightarrow \quad 4x \leq 88$ and $x \geq 8$

$\Rightarrow \quad x \leq 22$ and $x \geq 8$

$\Rightarrow \quad 8 \leq x \leq 22.$

Hence, the length of the shortest piece is to be greater than or equal to 8 but less than or equal to 22.

EXAMPLE 6 *In drilling world's deepest hole it was found that the temperature* $T(x)$ *in degree Celsius, x* km *below the earth's surface was given by* $T(x) = 30 + 25(x - 3)$, $3 \leq x \leq 15$. *At what depth will the temperature be between* 155°C *and* 205°C*?*

SOLUTION We have,

$T(x) = 30 + 25(x - 3)$

$\Rightarrow \quad T(x) = 30 + 25x - 75$

$\Rightarrow \quad T(x) = 25x - 45.$ … (i)

Let *x* km below the earth's surface, the temperature lies between 155°C and 205°C. Then,

$155 < T(x) < 205$

$\Rightarrow \quad 155 < 25x - 45 < 205$

$\Rightarrow \quad 155 < 25x - 45$ and $25x - 45 < 205$

$\Rightarrow \quad 155 + 45 < 25x \text{ and } 25x < 205 + 45$

$\Rightarrow \quad 200 < 25x \text{ and } 25x < 250$

$\Rightarrow \quad 8 < x \text{ and } x < 10$

$\Rightarrow \quad 8 < x < 10.$

Clearly, $8 < x < 10$ lies in the range $3 \le x \le 15$.

Hence, the required depth is more than 8 km and less than 10 km.

EXAMPLE 7 *A manufacturer has 460 litres of a 9% acid solution. How many litres of a 3% acid solution must be added to it so that the acid content in the resulting mixture be more than 5% but less than 7%?*

SOLUTION Let x litres of a 3% acid solution be added to 460 litres of 9% acid solution. Then,

total quantity of mixture $= (460 + x)$ litres.

Total acid content in $(460 + x)$ litres of mixture

$$= \left\{\left(460 \times \frac{9}{100}\right) + \left(x \times \frac{3}{100}\right)\right\} \text{ litres} = \left(\frac{207}{5} + \frac{3x}{100}\right) \text{ litres.}$$

Now, the acid content in the resulting mixture must be more than 5% and less than 7%.

$\therefore \quad 5\% \text{ of } (460 + x) < \left(\frac{207}{5} + \frac{3x}{100}\right) < 7\% \text{ of } (460 + x)$

$\Rightarrow \quad \frac{5}{100} \times (460 + x) < \frac{4140 + 3x}{100} < \frac{7}{100} \times (460 + x)$

$\Rightarrow \quad 5(460 + x) < 4140 + 3x < 7(460 + x)$

$\Rightarrow \quad 2300 + 5x < 4140 + 3x < 3220 + 7x$

$\Rightarrow \quad 2300 + 5x < 4140 + 3x \text{ and } 4140 + 3x < 3220 + 7x$

$\Rightarrow \quad 5x - 3x < 4140 - 2300 \text{ and } 4140 - 3220 < 7x - 3x$

$\Rightarrow \quad 2x < 1840 \text{ and } 920 < 4x$

$\Rightarrow \quad x < 920 \text{ and } 230 < x$

$\Rightarrow \quad 230 < x < 920.$

Hence, the required quantity of 3% acid solution to be added must be more than 230 litres and less than 920 litres.

EXAMPLE 8 *A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree Fahrenheit, if the conversion formula is* $F = \frac{9}{5}C + 32$?

SOLUTION $F = \frac{9}{5}C + 32 \Rightarrow C = \frac{5}{9}(F - 32).$ … (i)

$\therefore \quad 40 < C < 45$

$\Rightarrow \quad 40 < \frac{5}{9}(F - 32) < 45$

$\Rightarrow \quad 40 < \frac{5}{9}(F - 32) \text{ and } \frac{5}{9}(F - 32) < 45$

$\Rightarrow \quad 40 \times \frac{9}{5} < F - 32$ and $F - 32 < 45 \times \frac{9}{5}$

$\Rightarrow \quad 72 < F - 32$ and $F - 32 < 81$

$\Rightarrow \quad 72 + 32 < F$ and $F < 81 + 32$

$\Rightarrow \quad 104 < F$ and $F < 113$

$\Rightarrow \quad 104 < F < 113.$

Hence, the solution is to be kept between 104°F and 113°F.

EXAMPLE 9 *The IQ of a person is given by the formula,* $IQ = \frac{m}{c} \times 100$, *where m is the mental age and c is the chronological age. If* $80 \le IQ \le 140$ *for a group of 12-year children, find the range of their mental age.*

SOLUTION When $c = 12$, we have $IQ = \left(\frac{m}{12} \times 100\right) = \frac{25m}{3}$.

$\therefore \quad 80 \le IQ \le 140 \Rightarrow 80 \le \frac{25m}{3} \le 140$

$\Rightarrow 80 \le \frac{25m}{3}$ and $\frac{25m}{3} \le 140$

$\Rightarrow \frac{3}{25} \times 80 \le m$ and $m \le 140 \times \frac{3}{25}$

$\Rightarrow \frac{48}{5} \le m$ and $m \le \frac{84}{5}$

$\Rightarrow 9.6 \le m \le 16.8.$

Hence, the required mental age for a group of 12-year children is 9.6 years or more and 16.8 years or less.

EXERCISE 6A

1. Fill in the blanks with correct inequality sign $(>, <, \ge, \le)$.

(i) $5x < 20 \Rightarrow x \ldots\ldots 4$

(ii) $-3x > 9 \Rightarrow x \ldots\ldots -3$

(iii) $4x > -16 \Rightarrow x \ldots\ldots -4$

(iv) $-6x \le -18 \Rightarrow x \ldots\ldots 3$

(v) $x > -3 \Rightarrow -2x \ldots\ldots 6$

(vi) $a < b$ and $c < 0 \Rightarrow \frac{a}{c} \ldots\ldots \frac{b}{c}$

(vii) $p - q = -3 \Rightarrow p \ldots\ldots q$

(viii) $u - v = 2 \Rightarrow u \ldots\ldots v$

Solve each of the following inequations and represent the solution set on the number line.

2. $6x \le 25$, where (i) $x \in N$, (ii) $x \in Z$.

3. $-2x > 5$, where (i) $x \in Z$, (ii) $x \in R$.

4. $3x + 8 > 2$, where (i) $x \in Z$, (ii) $x \in R$.

5. $5x + 2 < 17$, where (i) $x \in Z$, (ii) $x \in R$.

6. $3x-4>x+6$, where $x\in R$.

7. $3-2x\geq 4x-9$, where $x\in R$.

8. $\frac{5x-8}{3}\geq\frac{4x-7}{2}$, where $x\in R$.

9. $\frac{5x}{4}-\frac{4x-1}{3}>1$, where $x\in R$.

10. $\frac{1}{4}\left(\frac{2}{3}x+1\right)\geq\frac{1}{3}(x-2)$, where $x\in R$.

11. $\frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$, where $x\in R$.

12. $\frac{x}{4}<\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$, where $x\in R$.

13. $\frac{(2x-1)}{3}\geq\frac{(3x-2)}{4}-\frac{(2-x)}{5}$, where $x\in R$.

Solve:

14. $\frac{x-3}{x+1}<0, x\in R$

15. $\frac{x-3}{x+4}>0, x\in R$

16. $\frac{2x-3}{3x-7}>0, x\in R$

17. $\frac{x-7}{x-2}\geq 0, x\in R$

18. $\frac{3}{x-2}>2, x\in R$

19. $\frac{1}{x-1}\leq 2, x\in R$

20. $\frac{5x+8}{4-x}<2, x\in R$

21. $|3x-7|>4, x\in R$

22. $|5-2x|\leq 3, x\in R$

23. $|4x-5|\leq\frac{1}{3}, x\in R$

24. $\frac{1}{|x|-3}\leq\frac{1}{2}, x\in R$

25. $\frac{|x+2|-x}{x}<2, x\in R$

26. $\left|\frac{2x-1}{x-1}\right|>2, x\in R$

27. $\frac{|x-3|}{x-3}>0, x\in R$

28. $\frac{|x|-1}{|x|-2}\geq 0, x\in R-\{-2,2\}$

29. $\frac{1}{2-|x|}\geq 1, x\in R-\{-2,2\}$

30. $|x+1|+|x|>3, x\in R$

31. $\left|\frac{2}{x-4}\right|>1, x\neq 4$

Solve the following systems of linear inequations:

32. $\frac{4}{x+1}\leq 3\leq\frac{6}{x+1}, x>0$

33. $-11\leq 4x-3\leq 13$

34. $5x-7<3(x+3), 1-\frac{3x}{2}\geq x-4$

35. $-2<\frac{6-5x}{4}<7$

36. $3x-2>x+\frac{4-x}{3}>3$

37. $\frac{7x-1}{2}<-3, \frac{3x+8}{5}+11<0$

38. $-12<4-\frac{3x}{-5}\leq 2$

39. $1\leq|x-2|\leq 3$

40. Find all pairs of consecutive odd positive integers, both of which are smaller than 18 such that their sum is more than 20.

41. Find all pairs of consecutive even positive integers both of which are larger than 8 such that their sum is less than 25.

42. A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$ respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold by the company to realise some profit?

43. The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of the pH values for the third reading that will result in the acidity level being normal.

44. A manufacturer has 640 litres of a 8% solution of boric acid. How many litres of a 2% boric acid solution be added to it so that the boric acid content in the resulting mixture will be more than 4% but less than 6%?

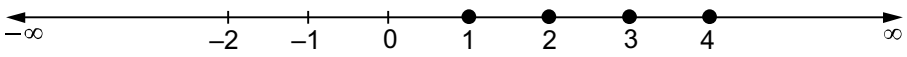
45. How many litres of water will have to be added to 600 litres of the 45% solution of acid so that the resulting mixture will contain more than 25%, but less than 30% acid content?

46. To receive grade A in a course one must obtain an average of 90 marks or more in five papers, each of 100 marks. If Tanvy scored 89, 93, 95 and 91 marks in first four papers, find the minimum marks that she must score in the last paper to get grade A in the course.

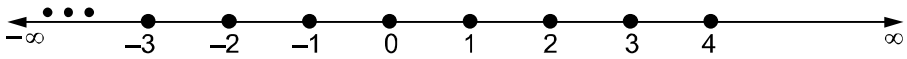
ANSWERS (EXERCISE 6A)

1. (i) $<$ (ii) $<$ (iii) $>$ (iv) $\geq$ (v) $<$ (vi) $>$ (vii) $<$ (viii) $>$

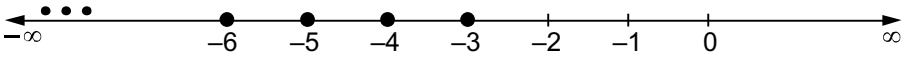
2. (i) {1, 2, 3, 4}



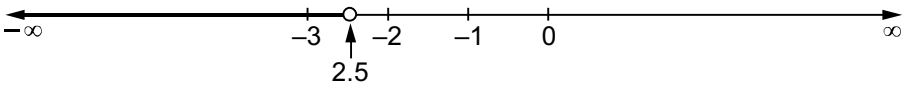
(ii) {…, –3, –2, –1, 0, 1, 2, 3, 4}



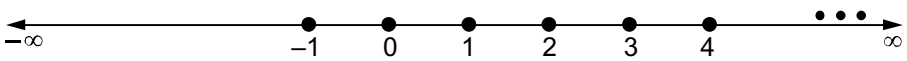
3. (i) {–3, –4, –5, –6, …}



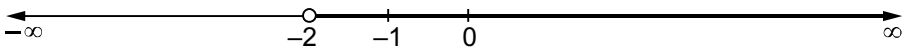
(ii) $(-\infty, -2.5)$



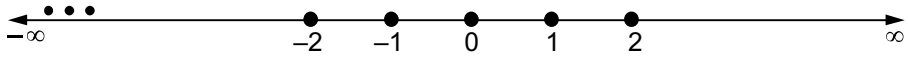
4. (i) {–1, 0, 1, 2, 3, 4, …}



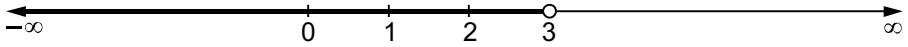
(ii) $(-2, \infty)$



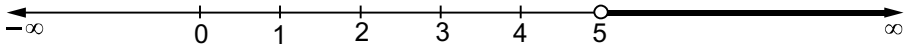
5. (i) $\{2, 1, 0, -1, -2, \ldots\}$



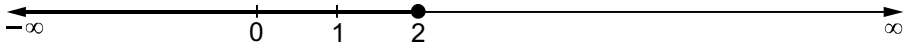
(ii) $(-\infty, 3)$



6. $(5, \infty)$

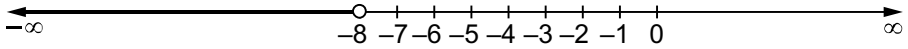


7. $(-\infty, 2]$

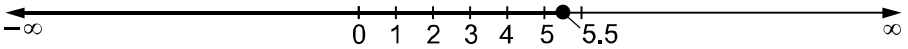


8. $\left(-\infty, \frac{5}{2}\right]$

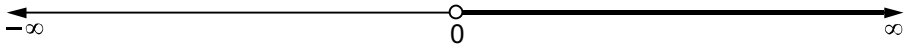
9. $(-\infty, -8)$



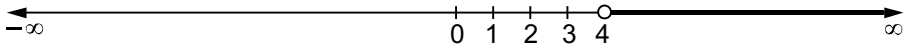
10. $(-\infty, 5.5]$



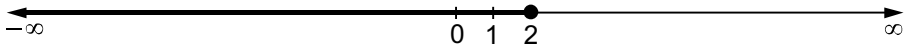
11. $(0, \infty)$



12. $(4, \infty)$



13. $(-\infty, 2]$



14. $(-1, 3)$

15. $(-\infty, -4) \cup (3, \infty)$

16. $\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{7}{3}, \infty\right)$

17. $(-\infty, 2) \cup [7, \infty)$

18. $\left(2, \frac{7}{2}\right)$

19. $(-\infty, 1) \cup \left[\frac{3}{2}, \infty\right)$

20. $(-\infty, 0) \cup (4, \infty)$

21. $(-\infty, 1) \cup \left(\frac{11}{3}, \infty\right)$

22. $[1, 4]$

23. $\left[\frac{7}{6}, \frac{4}{3}\right]$

24. $(-\infty, -5] \cup (-3, 3) \cup [5, \infty)$

25. $(-\infty, -2) \cup (1, \infty)$

26. $\left(\frac{3}{4}, 1\right) \cup (1, \infty)$

27. $(3, \infty)$

28. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

29. $(-2, -1] \cup [1, 2)$

30. $(-\infty, -2) \cup (1, \infty)$

31. $(2, 4) \cup (4, 6)$

32. $\frac{1}{3} \le x \le 1$

33. $-2 \le x \le 4$

34. $-\infty < x \le 2$

35. $-4.4 < x < 2.8$

36. $\frac{5}{2} < x < \infty$ **37.** $-\infty < x < -21$ **38.** $\frac{-80}{3} < x \le \frac{-10}{3}$

39. $[-1, 1] \cup [3, 5]$ **40.** (11, 13), (13, 15), (15, 17)

41. (10, 12) **42.** More than 2000 **43.** Between 7.77 and 8.67

44. More than 320 litres and less than 1280 litres

45. More than 300 litres but less than 480 litres **46.** 82 marks

HINTS TO SOME SELECTED QUESTIONS

1. (ii) $-3x > 9 \Rightarrow x < -3.$ (iii) $4x > -16 \Rightarrow x > -4.$

(iv) $-6x \le -18 \Rightarrow x \ge 3.$ (v) $x > -3 \Rightarrow -2x < 6.$

(vi) $a < b$ and $c < 0 \Rightarrow \frac{a}{c} > \frac{b}{c}.$ (vii) $p - q = -3 \Rightarrow p < q.$

3. $-2x > 5 \Rightarrow x < \frac{-5}{2} \Rightarrow x < -2.5.$

(i) All integers less than -2.5 are $-3, -4, -5, -6, \ldots$.

(ii) Set of all real numbers less than -2.5 are $(-\infty, -2.5)$.

4. $3x > -6 \Rightarrow x > -2.$

(i) All integers greater than -2 are $-1, 0, 1, 2, 3, 4, \ldots$.

(ii) Set of all real numbers greater than -2 is $(-2, \infty)$.

7. $3 - 2x \ge 4x - 9 \Rightarrow -6x \ge -12 \Rightarrow x \le 2.$

$\therefore$ solution set $= (-\infty, 2]$.

8. $10x - 16 \ge 12x - 21 \Rightarrow -2x \ge -5 \Rightarrow x \le \frac{5}{2}.$

$\therefore$ solution set $= \left(-\infty, \frac{5}{2}\right]$.

9. $15x - 4(4x - 1) > 12 \Rightarrow 15x - 16x + 4 > 12 \Rightarrow -x > 8 \Rightarrow x < -8.$

$\therefore$ solution set $= (-\infty, -8)$.

10. $\frac{1}{4} \cdot \frac{(2x+3)}{3} \ge \frac{x-2}{3} \Rightarrow 2x + 3 \ge 4x - 8 \Rightarrow -2x \ge -11 \Rightarrow x \le \frac{11}{2}.$

$\therefore$ solution set $= (-\infty, 5.5]$.

11. $(2x - 1) - 4(x - 1) < 3(3x + 1) \Rightarrow -2x + 3 < 9x + 3 \Rightarrow -11x < 0 \Rightarrow x > 0.$

$\therefore$ solution set $= (0, \infty)$.

12. $15x < 20(5x - 2) - 12(7x - 3) \Rightarrow 15x < 100x - 40 - 84x + 36$

$\Rightarrow 15x - 16x < -4 \Rightarrow -x < -4 \Rightarrow x > 4.$

$\therefore$ solution set $= (4, \infty)$.

13. $20(2x - 1) \ge 15(3x - 2) - 12(2 - x) \Rightarrow 40x - 20 \ge 45x - 30 - 24 + 12x$

$\Rightarrow -17x \ge -34 \Rightarrow x \le 2.$

$\therefore$ solution set $= (-\infty, 2]$.

14. $(x - 3 < 0$ and $x + 1 > 0)$ or $(x - 3 > 0$ and $x + 1 < 0)$

$\Rightarrow$ $(x < 3$ and $x > -1)$ or $(x > 3$ and $x < -1)$

$\Rightarrow$ $-1 < x < 3 \Rightarrow x \in (-1, 3).$ [$x \ge 3$ and $x < -1$ is not possible]

15. $(x-3<0 \text{ and } x+4<0)$ or $(x-3>0 \text{ and } x+4>0)$

$\Rightarrow$ $(x<3 \text{ and } x<-4)$ or $(x>3 \text{ and } x>-4)$

$\Rightarrow$ $(x<-4)$ or $(x>3) \Rightarrow x\in(-\infty,-4)\cup(3,\infty)$.

16. $(2x-3<0 \text{ and } 3x-7<0)$ or $(2x-3>0 \text{ and } 3x-7>0)$

$\Rightarrow$ $\left(x<\frac{3}{2} \text{ and } x<\frac{7}{3}\right)$ or $\left(x>\frac{3}{2} \text{ and } x>\frac{7}{3}\right)$

$\Rightarrow$ $\left(x<\frac{3}{2}\right)$ or $\left(x>\frac{7}{3}\right) \Rightarrow x\in\left(-\infty,\frac{3}{2}\right)\cup\left(\frac{7}{3},\infty\right)$.

17. $(x-7\le 0 \text{ and } x-2<0)$ or $(x-7\ge 0 \text{ and } x-2>0)$

$\Rightarrow$ $(x\le 7 \text{ and } x<2)$ or $(x\ge 7 \text{ and } x>2)$

$\Rightarrow$ $(x<2)$ or $(x\ge 7) \Rightarrow x\in(-\infty,2)\cup[7,\infty)$.

18. $\frac{3}{x-2}-2>0 \Rightarrow \frac{3-2x+4}{x-2}>0 \Rightarrow \frac{7-2x}{x-2}>0$

$\Rightarrow$ $(7-2x<0 \text{ and } x-2<0)$ or $(7-2x>0 \text{ and } x-2>0)$

$\Rightarrow$ $(7<2x \text{ and } x<2)$ or $(7>2x \text{ and } x>2)$

$\Rightarrow$ $\left(x>\frac{7}{2} \text{ and } x<2\right)$ or $\left(x<\frac{7}{2} \text{ and } x>2\right)$

$\Rightarrow$ $2<x<\frac{7}{2} \Rightarrow x\in\left(2,\frac{7}{2}\right)$. $\quad [x<\frac{7}{2} \text{ and } x>2 \text{ cannot hold}]$

19. $\frac{1}{x-1}-2\le 0 \Rightarrow \frac{1-2x+2}{x-1}\le 0 \Rightarrow \frac{3-2x}{x-1}\le 0$

$\Rightarrow$ $(3-2x\le 0 \text{ and } x-1>0)$ or $(3-2x\ge 0 \text{ and } x-1<0)$

$\Rightarrow$ $(3\le 2x \text{ and } x>1)$ or $(-2x\ge -3 \text{ and } x<1)$

$\Rightarrow$ $\left(x\ge\frac{3}{2} \text{ and } x>1\right)$ or $\left(x\le\frac{3}{2} \text{ and } x<1\right)$

$\Rightarrow$ $(x<1)$ or $\left(x\ge\frac{3}{2}\right)=x\in(-\infty,1)\cup\left[\frac{3}{2},\infty\right)$.

20. $\frac{5x+8}{4-x}-2<0 \Rightarrow \frac{5x+8-8+2x}{4-x}<0 \Rightarrow \frac{7x}{4-x}<0$

$\Rightarrow$ $(7x<0 \text{ and } 4-x>0)$ or $(7x>0 \text{ and } 4-x<0)$

$\Rightarrow$ $(x<0 \text{ and } 4>x)$ or $(x>0 \text{ and } -x<-4)$

$\Rightarrow$ $(x<0 \text{ and } x<4)$ or $(x>0 \text{ and } x>4)$

$\Rightarrow$ $(x<0)$ or $(x>4) \Rightarrow x\in(-\infty,0)\cup(4,\infty)$.

21. Using $|x|>a \Rightarrow x<-a$ or $x>a$, we get

$|3x-7|>4 \Rightarrow 3x-7<-4$ or $3x-7>4$

$\Rightarrow 3x<3$ or $3x>11 \Rightarrow x<1$ or $x>\frac{11}{3}$

$\Rightarrow x\in(-\infty,1)\cup\left(\frac{11}{3},\infty\right)$.

22. Using $|x|\le a \Leftrightarrow -a\le x\le a$, we get

$|5-2x|\le 3 \Rightarrow -3\le 5-2x\le 3$

$\Rightarrow -3\le 5-2x \text{ and } 5-2x\le 3$

$\Rightarrow 2x\le 8 \text{ and } 2x\ge 2 \Rightarrow x\le 4 \text{ and } x\ge 1$

$\Rightarrow 1\le x\le 4 \Rightarrow x\in[1,4]$.

23. Using $|x| \le a \Leftrightarrow -a \le x \le a$, we get

$$|4x-5| \le \frac{1}{3} \Rightarrow -\frac{1}{3} \le 4x-5 \le \frac{1}{3} \Rightarrow \frac{14}{3} \le 4x \text{ and } 4x \le \frac{16}{3}$$

$$\Rightarrow \frac{7}{6} \le x \text{ and } x \le \frac{4}{3} \Rightarrow \frac{7}{6} \le x \le \frac{4}{3} \Rightarrow x \in \left[\frac{7}{6}, \frac{4}{3}\right].$$

24. Case I *When* $x \ge 0$.

Then, $|x| = x$.

$$\therefore \quad \frac{1}{|x|-3} \le \frac{1}{2} \Rightarrow \frac{1}{x-3} - \frac{1}{2} \le 0 \Rightarrow \frac{2-x+3}{2(x-3)} \le 0 \Rightarrow \frac{5-x}{x-3} \le 0.$$

$\therefore \quad (5-x \le 0 \text{ and } x-3 > 0)$ or $(5-x \ge 0 \text{ and } x-3 < 0)$

$\Rightarrow \quad (x \ge 5 \text{ and } x > 3)$ or $(x \le 5 \text{ and } x < 3)$

$\Rightarrow \quad (x \ge 5)$ or $(x < 3)$

$\Rightarrow \quad (0 \le x < 3)$ or $(x \ge 5)$ $\qquad [\because\ x \ge 0]$

$\Rightarrow \quad x \in [0, 3) \cup [5, \infty)$.

Case II *When* $x < 0$.

Then, $|x| = -x$.

$$\therefore \quad \frac{1}{|x|-3} \le \frac{1}{2} \Rightarrow \frac{1}{-x-3} \le \frac{1}{2} \Rightarrow \frac{-1}{x+3} \le \frac{1}{2} \Rightarrow \frac{1}{2} + \frac{1}{x+3} \ge 0$$

$$\Rightarrow \frac{x+3+2}{x+3} \ge 0 \Rightarrow \frac{x+5}{x+3} \ge 0.$$

$\therefore \quad (x+5 \ge 0 \text{ and } x+3 > 0)$ or $(x+5 \le 0 \text{ and } x+3 < 0)$

$\Rightarrow \quad (x \ge -5 \text{ and } x > -3)$ or $(x \le -5 \text{ and } x < -3)$

$\Rightarrow \quad (x > -3)$ or $(x \le -5)$.

But, $x < 0$.

$\therefore \quad (-3 < x < 0)$ or $(x \le -5) \Rightarrow x \in (-3, 0) \cup (-\infty, -5]$.

Hence, $x \in (-\infty, -5] \cup (-3, 0) \cup [0, 3) \cup [5, \infty)$

$\Rightarrow \quad x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$.

25. $\dfrac{|x+2|-x}{x} - 2 < 0 \Rightarrow \dfrac{|x+2|-x-2x}{x} < 0 \Rightarrow \dfrac{|x+2|-3x}{x} < 0.$

Case I *When* $x+2 \ge 0$.

Then, $x \ge -2$ and $|x+2| = x+2$.

$$\therefore \quad \frac{|x+2|-3x}{x} < 0 \Rightarrow \frac{x+2-3x}{x} < 0 \Rightarrow \frac{2-2x}{x} < 0$$

$$\Rightarrow \frac{2}{x} - 2 < 0 \Rightarrow \frac{2}{x} < 2 \Rightarrow 2x > 2 \Rightarrow x > 1.$$

$\therefore \quad (x \ge -2 \text{ and } x > 1) \Rightarrow x > 1 \Rightarrow x \in (1, \infty)$.

Case II *When* $x+2 < 0$.

Then, $x < -2$ and $|x+2| = -(x+2)$.

$$\therefore \quad \frac{|x+2|-3x}{x} < 0 \Rightarrow \frac{-x-2-3x}{x} < 0 \Rightarrow \frac{-4x-2}{x} < 0$$

$$\Rightarrow \frac{4x+2}{x} > 0 \Rightarrow 4 + \frac{2}{x} > 0 \Rightarrow \frac{2}{x} > -4$$

$$\Rightarrow -4x < 2 \Rightarrow x < \frac{-1}{2}.$$

$$\therefore \quad x < -2 \text{ and } x < \frac{-1}{2} \Rightarrow x < -2 \Rightarrow x \in (-\infty, -2).$$

Hence, solution set $= (-\infty, -2) \cup (1, \infty)$.

26. Using $|x| > a \Leftrightarrow x < -a$ or $x > a$, we have

$$\left|\frac{2x-1}{x-1}\right| > 2 \Leftrightarrow \frac{2x-1}{x-1} < -2 \text{ or } \frac{2x-1}{x-1} > 2.$$

Case I *When* $\frac{2x-1}{x-1} < -2.$

Then, $\frac{2x-1}{x-1} + 2 < 0 \Rightarrow \frac{2x-1+2x-2}{x-1} < 0 \Rightarrow \frac{4x-3}{x-1} < 0.$

$\therefore \quad (4x-3 < 0 \text{ and } x-1 > 0) \text{ or } (4x-3 > 0 \text{ and } x-1 < 0)$

$\Rightarrow \quad \left(x < \frac{3}{4} \text{ and } x > 1\right) \text{ or } \left(x > \frac{3}{4} \text{ and } x < 1\right)$

$\Rightarrow \quad \frac{3}{4} < x < 1 \Rightarrow x \in \left(\frac{3}{4}, 1\right).$

Case II *When* $\frac{2x-1}{x-1} > 2.$

Then, $\frac{2x-1}{x-1} - 2 > 0 \Rightarrow \frac{2x-1-2x+2}{x-1} > 0 \Rightarrow \frac{1}{x-1} > 0$

$\Rightarrow x-1 > 0 \Rightarrow x > 1 \Rightarrow x \in (1, \infty).$

$\therefore \quad$ solution set $= \left(\frac{3}{4}, 1\right) \cup (1, \infty).$

27. $\frac{|x-3|}{x-3} > 0 \Leftrightarrow x-3 > 0 \Leftrightarrow x > 3 \Leftrightarrow x \in (3, \infty).$

28. Case I *When* $x \geq 0.$

Then, $|x| = x$ and so $\frac{|x|-1}{|x|-2} \geq 0 \Rightarrow \frac{x-1}{x-2} \geq 0.$

$\therefore \quad (x-1 \geq 0 \text{ and } x-2 > 0) \text{ or } (x-1 \leq 0 \text{ and } x-2 < 0)$

$\Rightarrow \quad (x \geq 1 \text{ and } x > 2) \text{ or } (x \leq 1 \text{ and } x < 2)$

$\Rightarrow \quad (x \leq 1) \text{ or } (x > 2) \Rightarrow (0 \leq x \leq 1) \text{ or } (x > 2) \qquad [\because x \geq 0]$

$\Rightarrow \quad x \in [0, 1] \cup (2, \infty).$

Case II *When* $x < 0.$

Then, $|x| = -x$ and so $\frac{|x|-1}{|x|-2} \geq 0 \Rightarrow \frac{-x-1}{-x-2} \geq 0 \Rightarrow \frac{x+1}{x+2} \geq 0.$

$\therefore \quad (x+1 \geq 0 \text{ and } x+2 > 0) \text{ or } (x+1 \leq 0 \text{ and } x+2 < 0)$

$\Rightarrow \quad (x \geq -1 \text{ and } x > -2) \text{ or } (x \leq -1 \text{ and } x < -2)$

$\Rightarrow \quad (x \geq -1) \text{ or } (x < -2)$

$\Rightarrow \quad (-1 \leq x < 0) \text{ or } (x < -2) \Rightarrow x \in [-1, 0) \cup (-\infty, -2). \qquad [\because x < 0]$

$\therefore \quad$ solution set $= [-1, 0) \cup [0, 1] \cup (-\infty, -2) \cup (2, \infty)$

$= [-1, 1] \cup (-\infty, -2) \cup (2, \infty).$

29. Case I *When* $x \geq 0.$

Then, $|x| = x$ and so $\frac{1}{2-|x|} \geq 1 \Rightarrow \frac{1}{2-x} - 1 \geq 0 \Rightarrow \frac{x-1}{2-x} \geq 0.$

$\therefore \quad (x-1 \geq 0 \text{ and } 2-x > 0) \text{ or } (x-1 \leq 0 \text{ and } 2-x < 0)$

$\Rightarrow \quad (x \geq 1 \text{ and } x < 2) \text{ or } (x \leq 1 \text{ and } x > 2)$

$\Rightarrow \quad (1 \leq x < 2) \Rightarrow x \in [1, 2).$

Case II *When* $x < 0.$

Then, $|x| = -x$. So, $\frac{1}{2-|x|} \geq 1 \Rightarrow \frac{1}{2+x} - 1 \geq 0 \Rightarrow \frac{-1-x}{2+x} \geq 0 \Rightarrow \frac{1+x}{2+x} \leq 0$

$\Rightarrow \quad (1+x \le 0 \text{ and } 2+x > 0) \text{ or } (1+x \ge 0 \text{ and } 2+x < 0)$

$\Rightarrow \quad (x \le -1 \text{ and } x > -2) \text{ or } (x \ge -1 \text{ and } x < -2)$

$\Rightarrow \quad (-2 < x \le -1) \Rightarrow x \in (-2, -1].$

$\therefore \quad$ solution set $= (-2, -1] \cup [1, 2).$

30. Putting $x+1=0$ and $x=0$, we get $x=-1$ and $x=0$ as the critical points. These points divide the whole real line into three parts, namely $(-\infty, -1), [-1, 0)$ and $[0, \infty)$.

Case I *When* $-\infty < x < -1$.

In this case, $x+1<0$ and $x<0$.

$\therefore \quad |x+1| = -(x+1) = -x-1$ and $|x| = -x$.

So, $|x+1|+|x|>3 \Rightarrow -x-1-x>3 \Rightarrow -2x>4 \Rightarrow x<-2$.

$\therefore \quad$ solution set in this case $= (-\infty, -2)$. $[\because -\infty < x < -1]$

Case II *When* $-1 \le x < 0$.

In this case, $x+1 \ge 0$ and $x<0$.

$\therefore \quad |x+1| = x+1$ and $|x| = -x$.

So, $|x+1|+|x|>3 \Rightarrow x+1-x>3 \Rightarrow 1>3$, which is absurd.

Case III *When* $0 \le x < \infty$.

In this case, $x+1>0$ and $x \ge 0$.

$\therefore \quad |x+1| = x+1$ and $|x| = x$.

So, $|x+1|+|x|>3 \Rightarrow x+1+x>3 \Rightarrow 2x>2 \Rightarrow x>1$.

$\therefore \quad$ solution set in this case $= (1, \infty)$.

Hence, solution set $= (-\infty, -2) \cup (1, \infty)$.

31. Using $|x|>a \Leftrightarrow x<-a \text{ or } x>a$, we get

$$\frac{2}{x-4} < -1 \text{ or } \frac{2}{x-4} > 1$$

$$\Rightarrow \quad \frac{2}{x-4}+1<0 \text{ or } \frac{2}{x-4}-1>0$$

$$\Rightarrow \quad \frac{x-2}{x-4}<0 \text{ or } \frac{6-x}{x-4}>0.$$

Case I *When* $\frac{x-2}{x-4}<0$.

Then, $(x-2<0 \text{ and } x-4>0) \text{ or } (x-2>0 \text{ and } x-4<0)$

$\Rightarrow \quad (x<2 \text{ and } x>4) \text{ or } (x>2 \text{ and } x<4)$

$\Rightarrow \quad 2<x<4 \Rightarrow x \in (2, 4).$

Case II *When* $\frac{6-x}{x-4}>0$.

Then, $(6-x<0 \text{ and } x-4<0) \text{ or } (6-x>0 \text{ and } x-4>0)$

$\Rightarrow \quad (x>6 \text{ and } x<4) \text{ or } (x<6 \text{ and } x>4)$

$\Rightarrow \quad 4<x<6 \Rightarrow x \in (4, 6).$

$\therefore \quad$ solution set $= (2, 4) \cup (4, 6).$

32. $\frac{4}{x+1} \le 3 \Rightarrow \frac{4}{x+1}-3 \le 0 \Rightarrow \frac{1-3x}{x+1} \le 0.$

$\therefore \quad (1-3x \le 0 \text{ and } x+1>0) \text{ or } (1-3x \ge 0 \text{ and } x+1<0)$

$\Rightarrow \quad \left(x \ge \frac{1}{3} \text{ and } x>-1\right) \text{ or } \left(x \le \frac{1}{3} \text{ and } x<-1\right)$

$\Rightarrow \quad x \geq \frac{1}{3}$ or $x < -1 \Rightarrow x \in \left[\frac{1}{3}, \infty\right) \cup (-\infty, -1)$

Again, $\frac{6}{x+1} \geq 3 \Rightarrow \frac{6}{x+1} - 3 \geq 0 \Rightarrow \frac{3-3x}{x+1} \geq 0 \Rightarrow \frac{1-x}{1+x} \geq 0.$

$\therefore \quad (1 - x \geq 0 \text{ and } 1 + x > 0)$ or $(1 - x \leq 0 \text{ and } 1 + x < 0)$

$\Rightarrow \quad (x \leq 1 \text{ and } x > -1)$ or $(x \geq 1 \text{ and } x < -1)$

$\Rightarrow \quad -1 < x \leq 1 \Rightarrow x \in (-1, 1].$

$\therefore \quad$ solution set $= \left\{(-\infty, -1) \cup \left[\frac{1}{3}, \infty\right)\right\} \cap (-1, 1] = \left[\frac{1}{3}, 1\right].$

39. (i) $|x-2| \geq 1 \Rightarrow (x-2) \leq -1$ or $(x-2) \geq 1$ $\quad [\because |x| \geq a \Rightarrow x \leq -a$ or $x \geq a]$

$\Rightarrow x \leq 1$ or $x \geq 3 \Rightarrow x \in (-\infty, 1] \cup [3, \infty).$

(ii) $|x-2| \leq 3 \Rightarrow -3 \leq x-2 \leq 3 \Rightarrow -1 \leq x \leq 5 \Rightarrow x \in [-1, 5].$

$\therefore \quad$ required solution set $= \{(-\infty, 1] \cup [3, \infty)\} \cap [-1, 5] = [-1, 1] \cup [3, 5].$

40. Let the required odd integers be x and $x+2$.

Then, $x + 2 < 18$ and $x + (x+2) > 20$

$\Rightarrow \quad x < 16$ and $2x + 2 > 20$

$\Rightarrow \quad x < 16$ and $x + 1 > 10$

$\Rightarrow \quad x > 9$ and $x < 16 \Rightarrow 9 < x < 16.$

So, the required values of x are 11, 13 and 15.

Hence, the required pairs are (11, 13), (13, 15) and (15, 17).

42. $P(x) = R(x) - C(x) \Rightarrow P(x) = 43x - (26000 + 30x)$

$\Rightarrow P(x) = (13x - 26000).$

Now, $P(x) > 0 \Rightarrow 13x - 26000 > 0 \Rightarrow x > 2000.$

Hence, more than 2000 cassettes must be sold.

43. Let the third reading be x. Then,

$$8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$$

$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$

$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83 \Rightarrow 7.77 < x < 8.67.$

44. Let x litres of 2% boric acid be added to 640 litres of 8% solution of boric acid. Then,

total mixture $= (640 + x)$ litres.

Boric acid content in this mixture $= \left(\frac{640 \times 8}{100} + \frac{x \times 2}{100}\right)$ litres.

$\therefore \quad \frac{4}{100} \times (640 + x) < \left\{\frac{640 \times 8}{100} + \frac{x \times 2}{100}\right\} < \frac{6}{100} \times (640 + x)$

$\Rightarrow \quad 2560 + 4x < 5120 + 2x < 3840 + 6x$

$\Rightarrow \quad 2x < 2560$ and $4x > 1280 \Rightarrow 320 < x < 1280.$

45. Let x litres of water may be added. Then,

$$25\% \text{ of } (600 + x) < \frac{600 \times 45}{100} < 30\% \text{ of } (600 + x)$$

$\Rightarrow \quad \frac{600 + x}{4} < 270 < \frac{1800 + 3x}{10}$

$\Rightarrow \quad \frac{600 + x}{4} < 270$ and $\frac{1800 + 3x}{10} > 270$

$\Rightarrow \quad 600 + x < 1080$ and $1800 + 3x > 2700 \Rightarrow 300 < x < 480.$

46. Let the required marks be x. Then,

$$(89+93+95+91+x) \geq (90 \times 5) \Rightarrow x \geq 82.$$

EXERCISE 6B

Very-Short-Answer Questions

1. Find the solution set of the inequation $\frac{1}{x-2} < 0$.

2. Find the solution set of the inequation $|x-1| < 2$.

3. Find the solution set of the inequation $|2x-3| > 1$.

4. Find the solution set of the inequation $\frac{|x-2|}{(x-2)} > 0, x \neq 2$.

5. Find the solution set of the inequation $\frac{x+1}{x+2} > 1$.

6. Solve the system of inequations $x-2 \geq 0, 2x-5 \leq 3$.

7. Solve $-4x > 16$, when $x \in Z$.

8. Solve $x+5 > 4x-10$, when $x \in R$.

9. Solve $\frac{3}{x-2} < 1$, when $x \in R$.

10. Solve $\frac{x}{x-5} > \frac{1}{2}$, when $x \in R$.

11. Solve $|x| < 4$, where $x \in R$.

12. Solve $|x| > 4$, where $x \in R$.

ANSWERS (EXERCISE 6B)

1. $(-\infty, 2)$ **2.** $(-1, 3)$ **3.** $(-\infty, 1) \cup (2, \infty)$ **4.** $(2, \infty)$

5. $(-\infty, -2)$ **6.** $x \in [2, 4]$ **7.** $x \in \{-5, -6, -7, \ldots\}$ **8.** $x \in (-\infty, 5)$

9. $x \in (-\infty, 2) \cup (5, \infty)$ **10.** $x \in (-\infty, -5) \cup (5, \infty)$ **11.** $x \in (-4, 4)$

12. $x \in (-\infty, -4) \cup (4, \infty)$

HINTS TO SOME SELECTED QUESTIONS

1. $\frac{1}{(x-2)} < 0 \Rightarrow x-2 < 0 \Rightarrow x < 2 \Rightarrow x \in (-\infty, 2)$.

2. Using $|x| < a \Leftrightarrow -a < x < a$, we get

$$|x-1| < 2 \Rightarrow -2 < x-1 < 2 \Rightarrow -1 < x < 3 \Rightarrow x \in (-1, 3).$$

3. Using $|x| > a \Leftrightarrow x < -a$ or $x > a$, we get

$$|2x-3| > 1 \Rightarrow 2x-3 < -1 \text{ or } 2x-3 > 1$$
$$\Rightarrow x < 1 \text{ or } x > 2 \Rightarrow x \in (-\infty, 1) \cup (2, \infty).$$

4. Clearly, $(x-2) > 0 \Rightarrow x > 2 \Rightarrow x \in (2, \infty)$.

5. $\frac{x+1}{x+2}-1>0 \Rightarrow \frac{x+1-x-2}{x+2}>0 \Rightarrow \frac{-1}{x+2}>0 \Rightarrow x+2<0$

$\Rightarrow x<-2 \Rightarrow x\in(-\infty,-2).$

6. $(x-2\geq 0 \text{ and } 2x-5\leq 3)$

$\Rightarrow \quad (x\geq 2 \text{ and } x\leq 4) \Rightarrow 2\leq x\leq 4 \Rightarrow x\in[2,4].$

7. $-4x>16 \Rightarrow x<-4 \Rightarrow x\in[-5,-6,-7,-8,\ldots\}.$

8. $x+5>4x-10 \Rightarrow 3x<15 \Rightarrow x<5 \Rightarrow x\in(-\infty,5).$

9. $\frac{3}{x-2}-1<0 \Rightarrow \frac{3-x+2}{x-2}<0 \Rightarrow \frac{5-x}{x-2}<0.$

$\therefore \quad (5-x<0 \text{ and } x-2>0) \text{ or } (5-x>0 \text{ and } x-2<0)$

$\Rightarrow \quad (x>5 \text{ and } x>2) \text{ or } (x<5 \text{ and } x<2)$

$\Rightarrow \quad (x>5 \text{ or } x<2) \Rightarrow x\in(-\infty,2)\cup(5,\infty).$

10. $\frac{x}{x-5}-\frac{1}{2}>0 \Rightarrow \frac{2x-x+5}{2(x-5)}>0 \Rightarrow \frac{x+5}{x-5}>0.$

$\therefore \quad (x+5<0 \text{ and } x-5<0) \text{ or } (x+5>0 \text{ and } x-5>0)$

$\Rightarrow \quad (x<-5 \text{ and } x<5) \text{ or } (x>-5 \text{ and } x>5)$

$\Rightarrow \quad (x<-5) \text{ or } (x>5) \Rightarrow x\in(-\infty,-5)\cup(5,\infty).$

11. Using $|x|<a \Leftrightarrow -a<x<a$, we get

$|x|<4 \Rightarrow -4<x<4 \Rightarrow x\in(-4,4).$

12. Using $|x|>a \Leftrightarrow x<-a \text{ or } x>a$, we get

$|x|>4 \Leftrightarrow x<-4 \text{ or } x>4$

$\Leftrightarrow x\in(-\infty,-4)\cup(4,\infty).$

KEY FACTS AND FORMULAE

1. (i) **Linear Inequations in One Variable:**

Inequalities of the form:

(i) $ax+b<c$ (ii) $ax+b\leq c$ (iii) $ax+b>c$ (iv) $ax+b\geq c$

where $a, b, c\in R$, $a\neq 0$ and x is a variable, are called inequations in x.

(ii) **Replacement Set** A set from which the values are replaced in an inequation in x, is called the replacement set.

(iii) **Solution Set** The set of all those values of x taken from the replacement set which satisfy the given inequation is called the solution set of the inequation.

2. Rules for Solving an Inequation

RULE 1 Adding the same number or expression to each side of an inequation does not change the inequality.

RULE 2 Subtracting the same number or expression from each side of an inequation does not change the inequality.

RULE 3 Multiplying (or dividing) each side of an inequation by the same positive number does not change the inequality.

RULE 4 Multiplying (or dividing) each side of an inequation by the same negative number reverses the inequality.

Thus, $-x > 3 \Rightarrow x < -3$ [on multiplying both sides by –1].

And, $-3x \leq -12 \Rightarrow x \geq 4$ [on dividing both sides by –3].

TRANSPOSITION Using Rule 1 and Rule 2, we can drop any term from one side of an inequation and put it on the other side with the opposite sign.

3. Some Useful Results

Let a be a positive real number. Then,

(i) $|x| < a \Leftrightarrow -a < x < a \Leftrightarrow x \in (-a, a)$.

(ii) $|x| \leq a \Leftrightarrow -a \leq x \leq a \Leftrightarrow x \in [-a, a]$.

(iii) $|x| > a \Leftrightarrow x < -a$ or $x > a$.

(iv) $|x| \geq a \Leftrightarrow x \leq -a$ or $x \geq a$.

4. Solving Two Simultaneous Linear Inequations in *x*

Suppose we have to solve two inequations in x simultaneously. Then, we find the solution set for each one of them. The intersection of these solution sets is the required solution set.

Linear Inequations (In two variables)

LINEAR INEQUATIONS *The statements of inequalities of the form* $ax + by + c > 0$, $ax + by + c \geq 0$, $ax + by + c < 0$ *and* $ax + by + c \leq 0$ *are called linear inequations in two variables x and y.*

The set of all ordered pairs (x, y) which satisfy the given inequation is called the solution set of the inequation.

GRAPH OF A LINEAR INEQUATION Let an inequation of any of the types given above be given.

Then, in order to solve it, we proceed according to steps given below.

Step 1 Consider the equation, $ax + by + c = 0$.

Draw the graph of this equation, which is a line.

In case of strict inequation > or < draw the line dotted, otherwise make it thick.

This line divides the plane into two equal parts.

Step 2 Choose a point [*if possible* (0, 0)], not lying on this line. If this point satisfies the given inequation then shade the part of the plane containing this point, otherwise shade the other part.

The shaded portion represents the solution set of the given inequation.

The dotted line is not a part of the solution, while the thick line is a part of it.

SOLVED EXAMPLES

EXAMPLE 1 *Solve* $3x + 4y \leq 12$ *graphically.*

SOLUTION Consider the equation, $3x + 4y = 12$.

The values of (x, y) satisfying $3x + 4y = 12$ are:

x	4	0
y	0	3

Plot the points $A(4, 0)$ and $B(0, 3)$ on a graph paper.

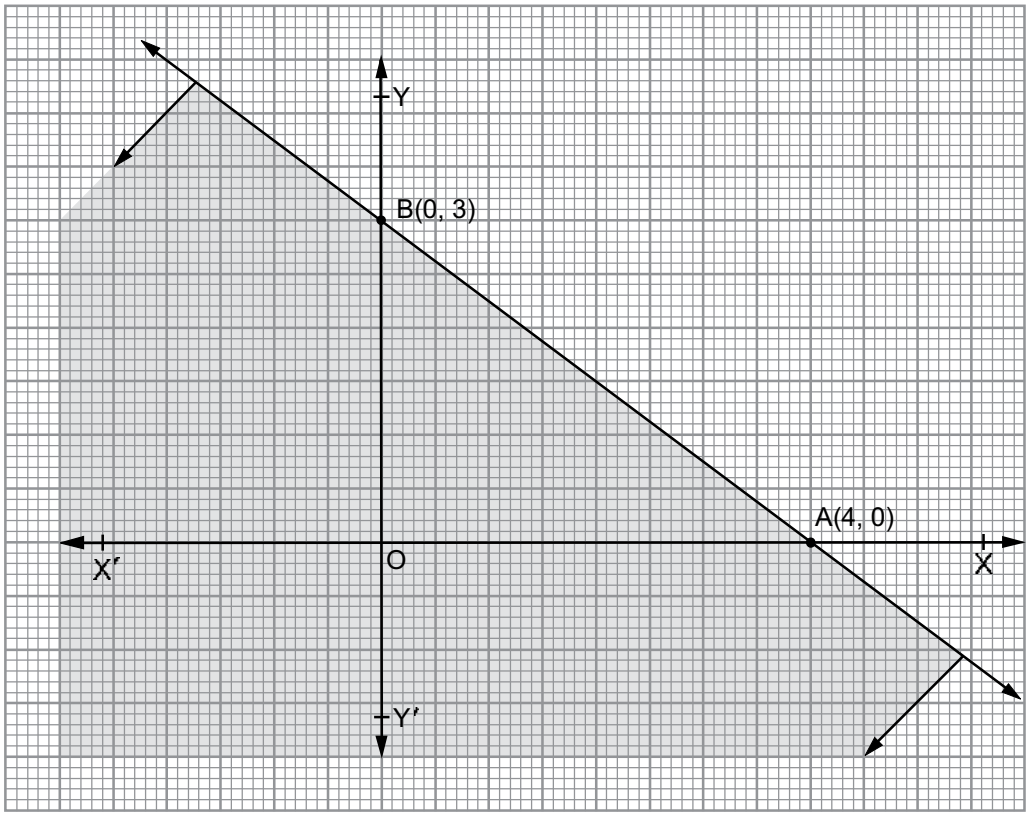
Then, line AB represents $3x + 4y = 12$.

This line divides the plane of the paper in two equal parts.

Clearly, the point (0, 0) satisfies $3x + 4y \leq 12$.

So, we shade that part of the plane divided by AB which contains (0, 0).

The shaded part of the plane together with all the points on line AB represents $3x + 4y \leq 12$.



EXAMPLE 2 *Draw the graph of the solution set of the inequation* $2x - y \geq 1$.

SOLUTION Consider the equation, $2x - y = 1$.

The values of (x, y) satisfying $2x - y = 1$ are:

x	2	0
y	3	–1

Take a graph paper and plot the points $A(2, 3)$ and $B(0, -1)$.

Then, line AB represents $2x - y = 1$.

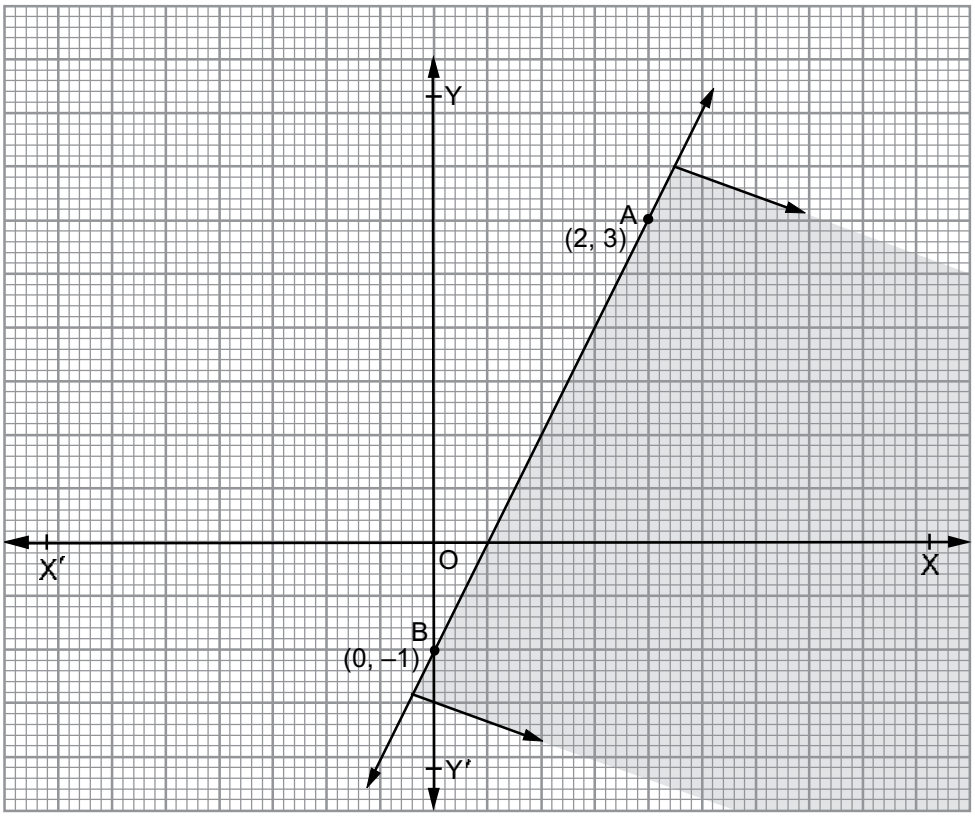
This line divides the plane of the paper in two equal parts.

Clearly, the point (0, 0) does not lie on $2x - y = 1$.

Also, (0, 0) does not satisfy $2x - y \geq 1$.

So, we shade that part of the plane divided by line AB which does not contain (0, 0), as shown below.

The shaded part of the plane together with all points on the line AB constitutes the solution set of the inequation, $2x - y \geq 1$.



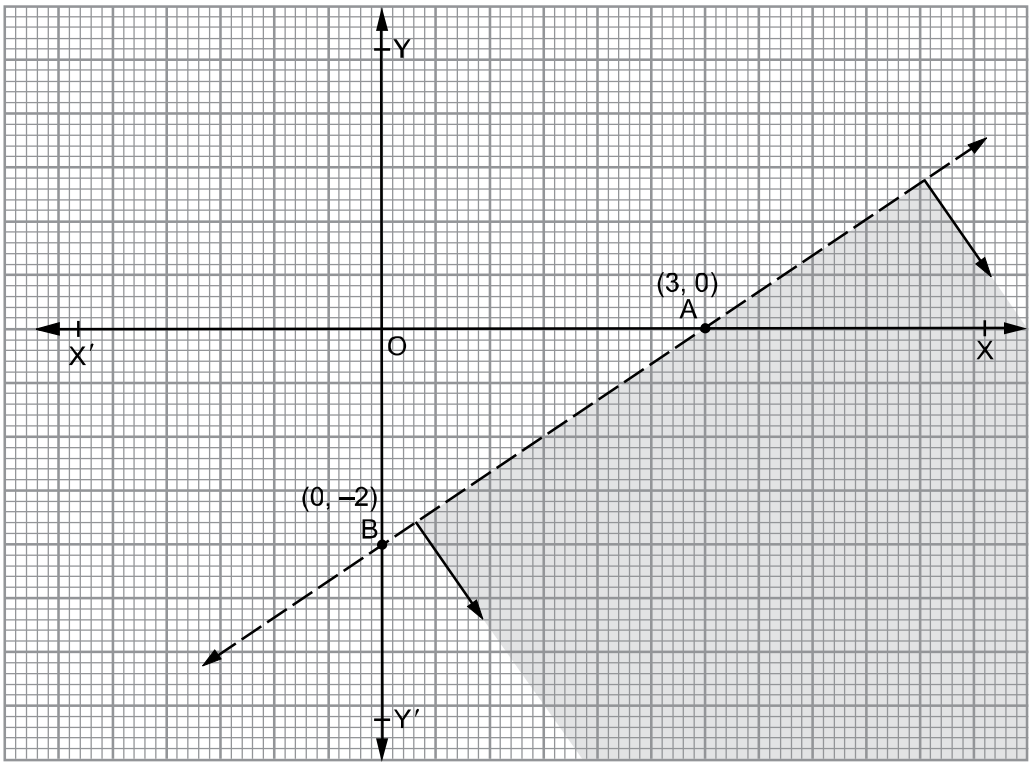
EXAMPLE 3 *Solve* $2x - 3y > 6$ *graphically.*

SOLUTION Consider the equation, $2x - 3y = 6$.

This equation may be written as, $\frac{x}{3} + \frac{y}{-2} = 1$.

Thus, it cuts the *x*-axis at $A(3, 0)$ and the *y*-axis at $B(0, -2)$.

$\therefore$ *AB* represents the line, $2x - 3y = 6$.



Clearly, the point (0, 0) satisfies $2x - 3y < 6$.

$\therefore$ (0, 0) does not satisfy $2x - 3y > 6$.

So, we shade that part of the plane divided by *AB* which does not contain $O(0, 0)$.

The shaded part of the plane excluding line *AB* forms the solution set of $2x - 3y > 6$, as shown above.

EXAMPLE 4 *Solve* $x + y < 5$ *graphically.*

SOLUTION Consider the equation, $x + y = 5$.

This equation may be written as $\frac{x}{5} + \frac{y}{5} = 1$.

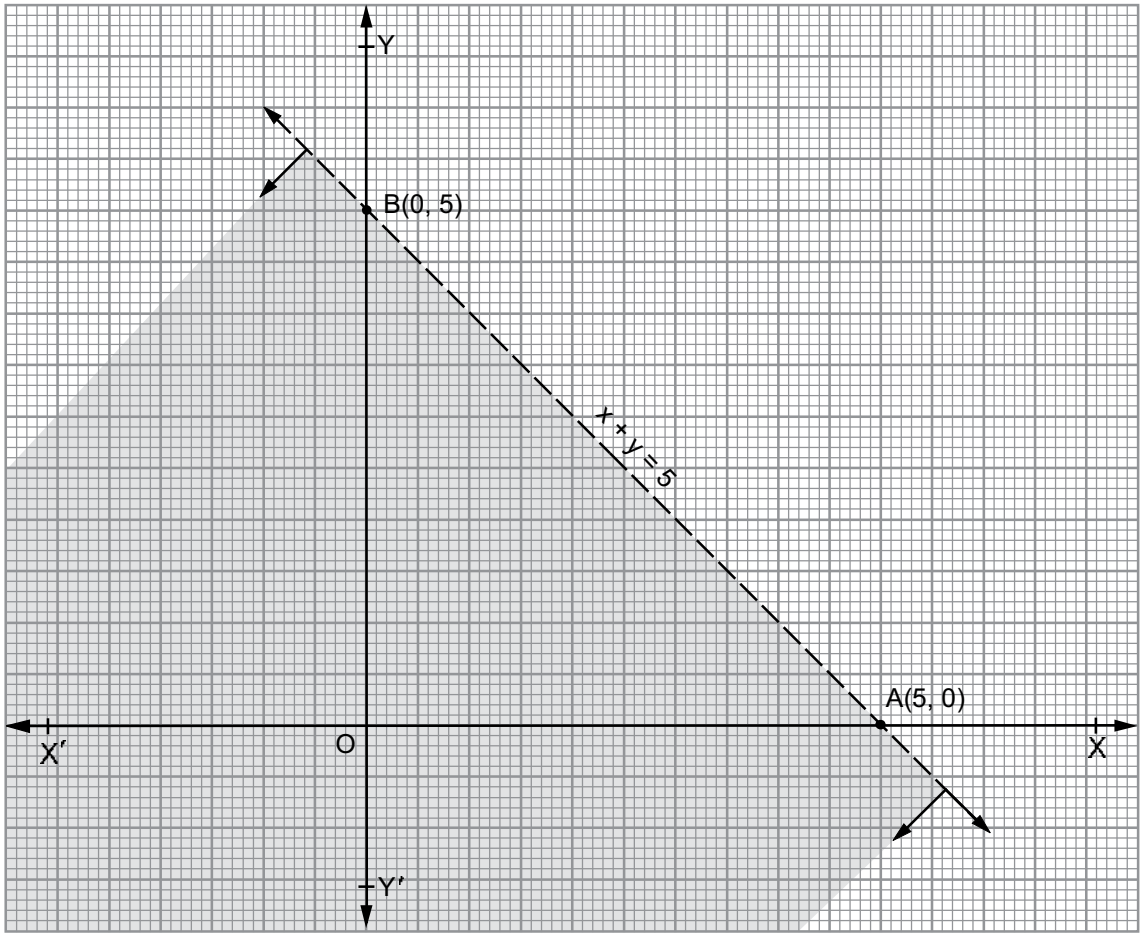
Thus, it cuts the *x*-axis at $A(5, 0)$ and the *y*-axis at $B(0, 5)$.

$\therefore$ *AB* represents the line $x + y = 5$.

Clearly, (0, 0) satisfies the inequation $x + y < 5$.

So, we shade that part of the plane divided by *AB* which contains $O(0, 0)$.

The shaded part of the plane excluding line *AB* forms the solution set of $x + y < 5$, as shown below.



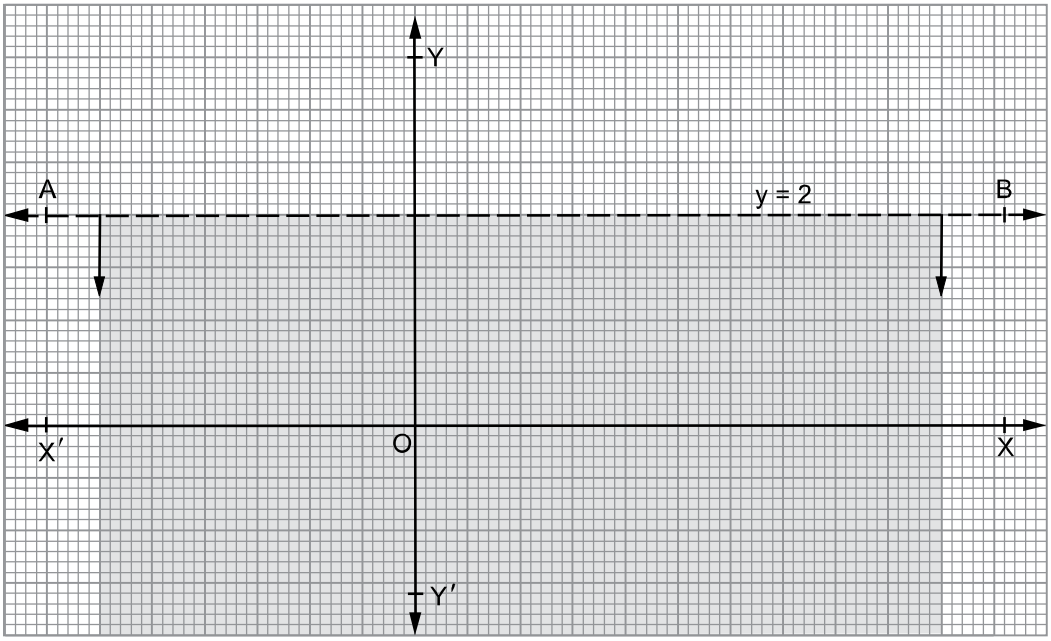
EXAMPLE 5 *Solve* $y < 2$ *graphically.*

SOLUTION The graph of $y = 2$ is a straight line *AB* drawn at a distance of 2 units above the *x*-axis and parallel to the *x*-axis.

Clearly, the point (0, 0) satisfies the inequation, $y < 2$.

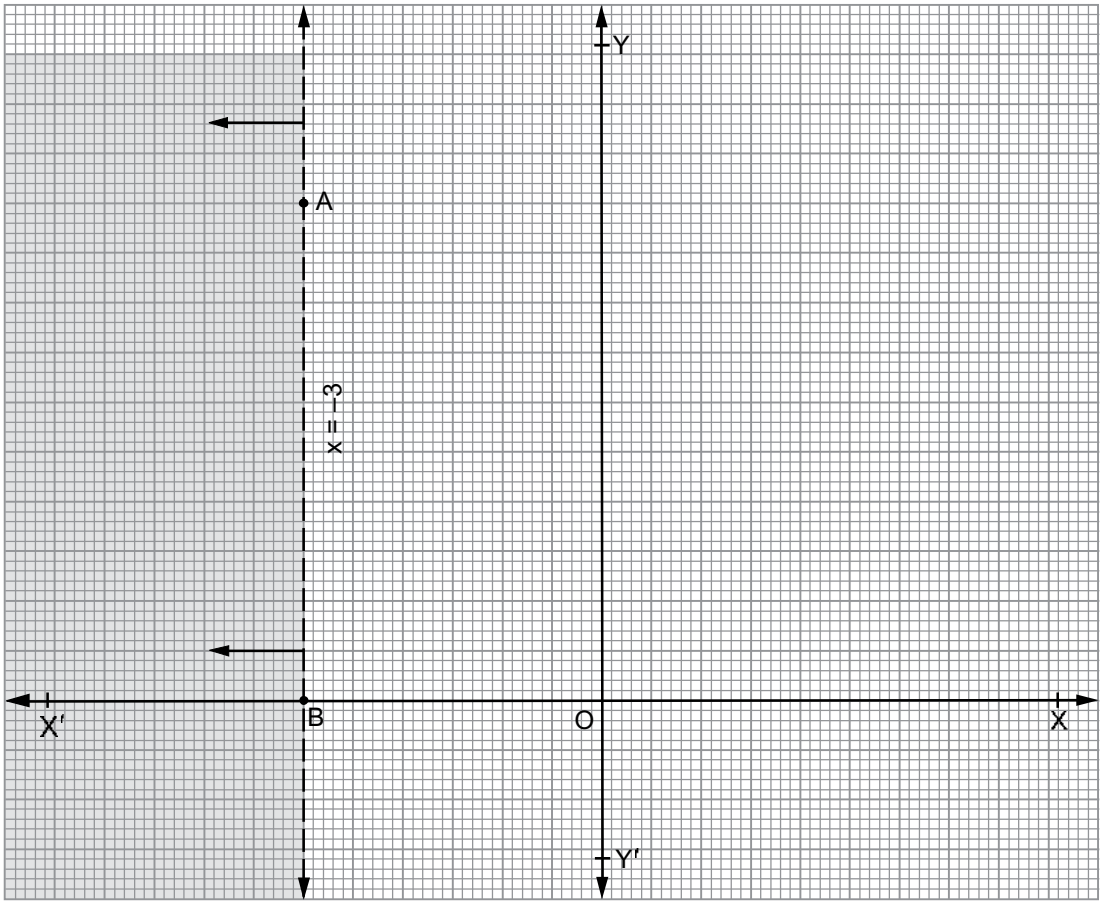
So, we shade that part of the plane divided by AB which contains the origin $O(0, 0)$, excluding AB.

The shaded part of the plane forms the solution set of the inequation, $y < 2$, as shown below.



EXAMPLE 6 *Solve the inequation* $x < -3$ *graphically.*

SOLUTION We know that the graph of $x = -3$ is a straight line AB drawn parallel to the y-axis at a distance of 3 units to the left of the y-axis.



Clearly, the point (0, 0) does not satisfy the inequation, $x < -3$.

So, we shade that part of the plane divided by AB which does not contain $O(0, 0)$, excluding AB.

The shaded part of the plane is the solution set of $x < -3$, as shown above.

EXAMPLE 7 *Exhibit graphically the solution set of the linear inequations:*

$$x + 2y \leq 10,\ x + y \leq 6,\ x \leq 4,\ x \geq 0 \text{ and } y \geq 0.$$

SOLUTION (i) First, we draw the graph of $x + 2y \leq 10$.

Consider the line, $x + 2y = 10$.

Clearly, the points $A(0, 5)$ and $B(10, 0)$ satisfy, $x + 2y = 10$.

Then, line AB represents $x + 2y = 10$.

Clearly, (0, 0) satisfies the inequation $x + 2y \leq 10$.

Thus, the line AB and part of the plane containing $O(0, 0)$ represent the solution set of $x + 2y \leq 10$.

(ii) Next, we draw the graph of $x + y \leq 6$.

Consider the line, $x + y = 6$.

Clearly, the points $C(0, 6)$ and $D(6, 0)$ satisfy $x + y = 6$.

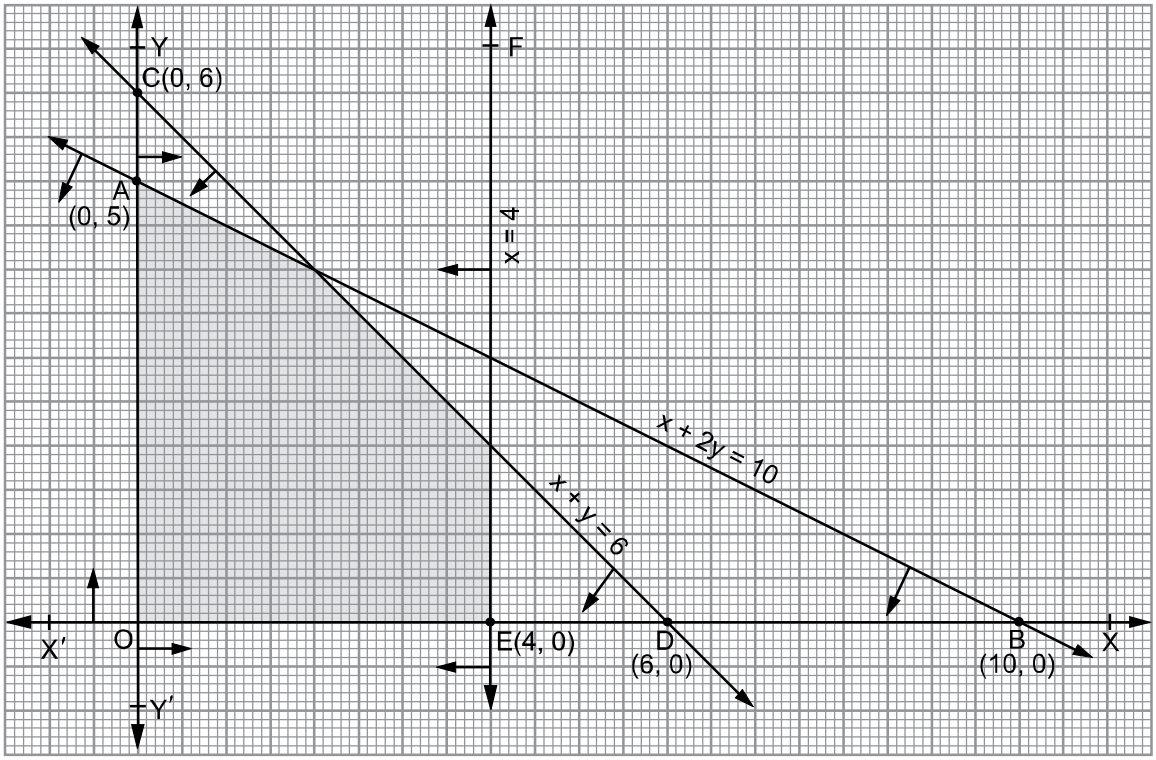
So, line CD represents $x + y = 6$.

Clearly, (0, 0) satisfies the inequation, $x + y \leq 6$.

So, the line CD and part of the plane containing $O(0, 0)$ represent the solution set of $x + y \leq 6$.

(iii) Next, we draw the graph of $x \leq 4$.

We know that $x = 4$ is the line EF parallel to the y-axis and passing through the point $E(4, 0)$.



Clearly, (0, 0) satisfies the inequation, $x \leq 4$.

Thus, the line *EF* and part of the plane containing *O*(0, 0) represent the solution set of $x \leq 4$.

(iv) $x \geq 0$ is represented by the *y*-axis and the plane on its right.

(v) $y \geq 0$ is represented by the *x*-axis and the plane above the *x*-axis.

The intersection of all these planes is the shaded part, which together with its boundary represents the solution of the given system of inequations.

EXAMPLE 8 *Draw the graph of the solution set of the inequations:*

$$2x + y \geq 2,\ x - y \leq 1,\ x + 2y \leq 8,\ x \geq 0 \text{ and } y \geq 0.$$

SOLUTION (i) First, we draw the graph of $2x + y \geq 2$.

Consider the line, $2x + y = 2$.

Now, $2x + y = 2 \Rightarrow \frac{x}{1} + \frac{y}{2} = 1$.

This line meets the axes at *A*(1, 0) and *B*(0, 2).

Clearly, the line *AB* represents $2x + y = 2$.

Now, (0, 0) does not satisfy $2x + y \geq 2$.

Thus, the line *AB* and part of the plane separated by *AB* and not containing *O*(0, 0), represent the solution set of $2x + y \geq 2$.

(ii) Next, we draw the graph of $x - y \leq 1$.

Consider the line, $x - y = 1$.

$$x - y = 1 \Rightarrow \frac{x}{1} + \frac{y}{(-1)} = 1.$$

This line meets at the axes at *A*(1, 0) and *C*(0, –1).

Clearly, the line *AC* represents $x - y = 1$.

Also, (0, 0) satisfies, $x - y \leq 1$.

Thus, the line *AC* and part of the plane separated by *AC* and containing *O*(0, 0) represent the solution set of $x - y \leq 1$.

(iii) Now, we draw the graph of $x + 2y \leq 8$.

Consider the line, $x + 2y = 8$.

Now, $x + 2y = 8 \Rightarrow \frac{x}{8} + \frac{y}{4} = 1$.

This line meets the axes at *D*(8, 0) and *E*(0, 4).

So, the line *DE* represents $x + 2y = 8$.

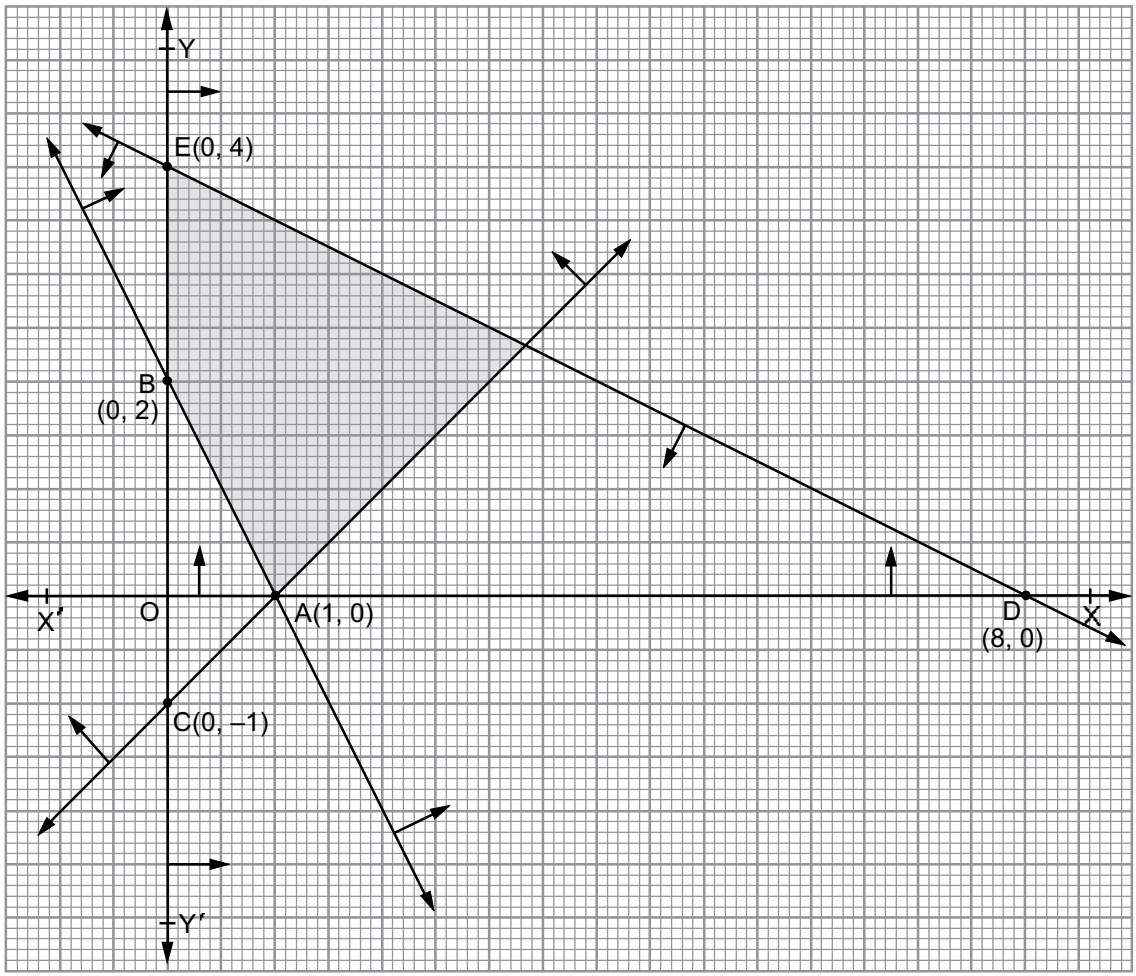
Also, (0, 0) satisfies the inequation, $x + 2y \leq 8$.

Thus, the line *DE* and part of the plane containing *O*(0, 0) represent the solution set of $x + 2y \leq 8$.

(iv) $x \geq 0$ is represented by the *y*-axis and the plane on its right.

(v) $y \geq 0$ is represented by the *x*-axis and the plane above the *x*-axis.

The intersection of all these planes is the required shaded part, representing the solution of the given system.



EXAMPLE 9 *Solve the following system of inequations graphically:*

$5x + 4y \leq 40,\ x \geq 2,\ y \geq 3.$

SOLUTION (i) First, we draw the graph line $5x + 4y = 40$.

Now, $5x + 4y = 40 \Rightarrow \frac{x}{8} + \frac{y}{10} = 1.$

This line meets the axes at $A(8, 0)$ and $B(0, 10)$.

Clearly, the line *AB* represents $5x + 4y = 40$.

Now, (0, 0) satisfies the inequation $5x + 4y \leq 40$.

Thus, the line *AB* and part of the plane containing *O*(0, 0) and separated by *AB* represent the solution set of the inequation, $5x + 4y \leq 40$.

(ii) Clearly, $x = 2$ is the line *CD* parallel to the *y*-axis at a distance of 2 units to its right.

Clearly, (0, 0) does not satisfy the inequation, $x \geq 2$.

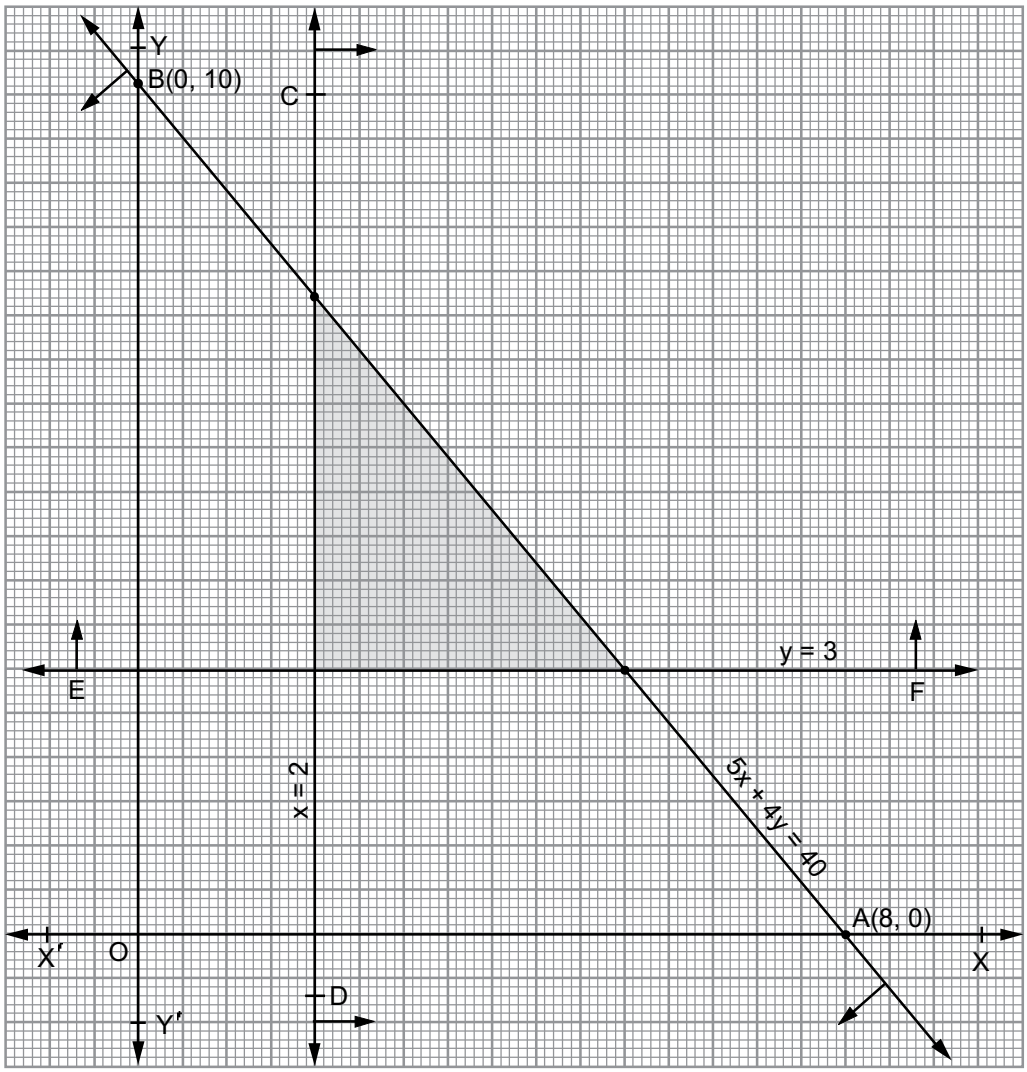
So, $x \geq 2$ consists of the line *CD* and part of the plane on right of *CD*.

(iii) Also, $y = 3$ is the line *EF* parallel to the *x*-axis lying above it at a distance of 3 units from it.

Clearly, (0, 0) does not satisfy the inequation, $y \geq 3$.

So, $y \geq 3$ consists of the line *EF* and part of the plane above it.

The intersection of all these planes is the required shaded part, representing the solution of the given system, as shown below.



EXAMPLE 10 *Show that the following system of linear inequations has no solution:*

$x + 2y \leq 3, 3x + 4y \geq 12, x \geq 0, y \geq 1.$

SOLUTION (i) First, we draw the graph of $x + 2y \leq 3$.

Consider the line, $x + 2y = 3$.

Clearly, the points $A(1, 1)$ and $B(-1, 2)$ lie on this line.

Then, line AB represents $x + 2y = 3$.

Now, (0, 0) satisfies $x + 2y \leq 3$.

So, the line AB and part of the plane containing $O(0,0)$ represent the solution set of $x + 2y \leq 3$.

(ii) Next, we draw the graph of $3x + 4y \geq 12$.

Consider the line, $3x + 4y = 12$.

Now, $3x + 4y = 12 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1.$

So, this line meets the axes at $C(4, 0)$ and $D(0, 3)$.

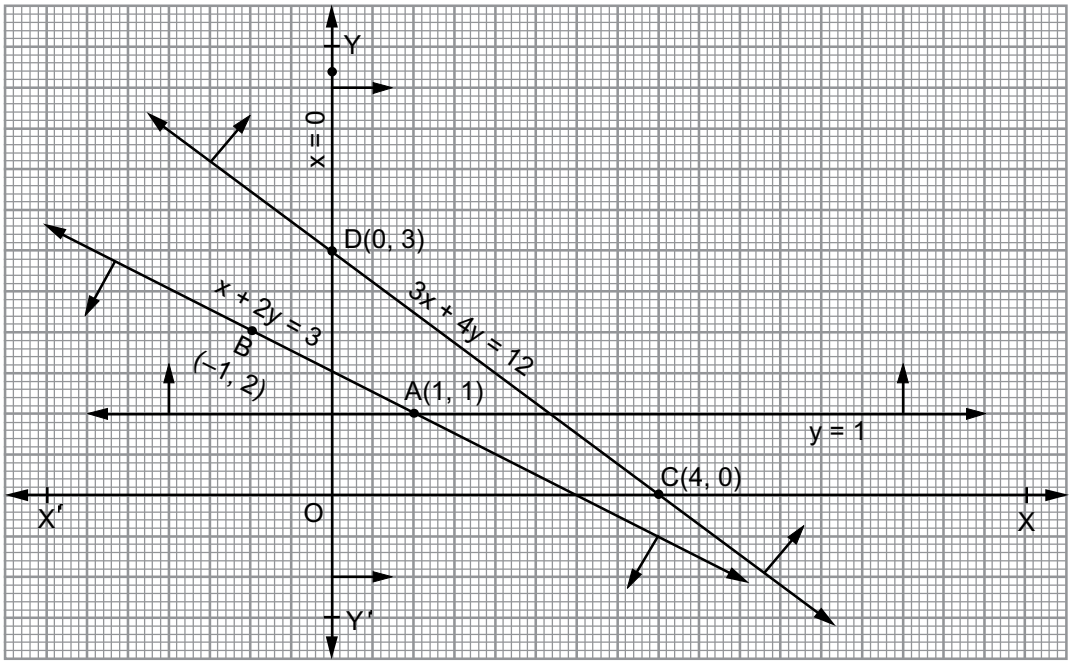
Thus, CD represents the line $3x + 4y \geq 12$.

Clearly, $(0, 0)$ does not satisfy $3x + 4y \geq 12$.

Therefore, the line CD and part of the plane not containing $O(0, 0)$ represent the solution set of $3x + 4y \geq 12$.

(iii) Clearly, $x \geq 0$ is represented by the y-axis and the plane on its right.

(iv) $y \geq 1$ represents a line parallel to the x-axis at a distance of 1 unit above the x-axis and the plane above this line.



Clearly, all these planes have no common intersection.

Hence, the given system of inequations has no solution.

EXAMPLE 11 *Show that the solution set of the following linear inequations is an unbounded region* $2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$.

SOLUTION (i) First, we draw the graph of $2x + y \geq 8$.

Consider the line, $2x + y = 8$.

Now, $2x + y = 8 \Rightarrow \frac{x}{4} + \frac{y}{8} = 1$.

This line meets the axes at $A(4, 0)$ and $B(0, 8)$.

Clearly, the line AB represents $2x + y = 8$.

It is also clear that $(0, 0)$ does not satisfy $2x + y \geq 8$.

Thus, the line AB and the part of the plane not containing $O(0, 0)$ represent the solution of the inequation $2x + y \geq 8$.

(ii) Next, we draw the graph of $x + 2y \geq 10$.

Consider the line, $x + 2y = 10$.

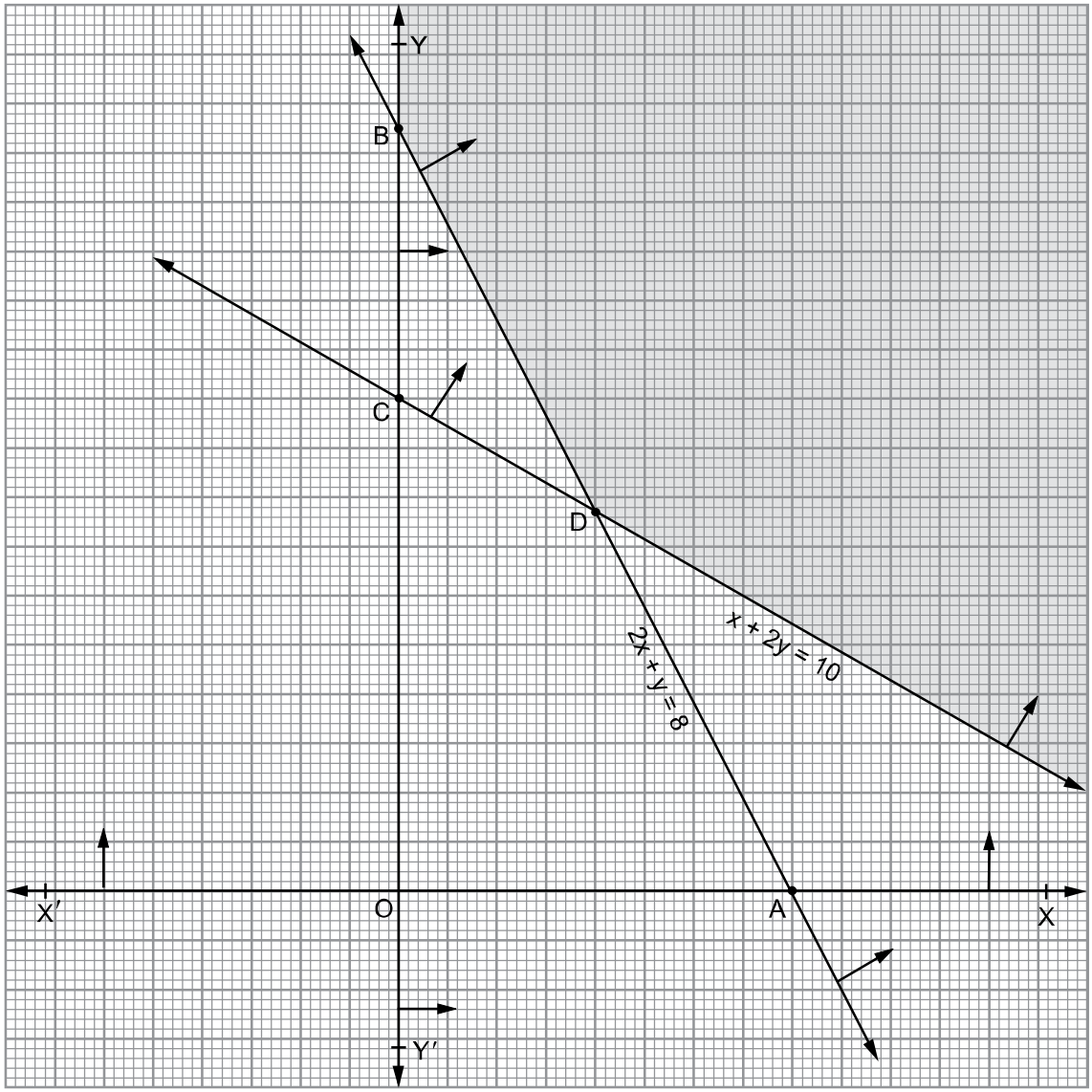
Clearly, the points $C(0, 5)$ and $D(2, 4)$ lie on this line.

Also, $(0, 0)$ does not satisfy $x + 2y \geq 10$.

So, the line CD and the part of the plane not containing $O(0, 0)$ represent the solution set of $x + 2y \geq 10$.

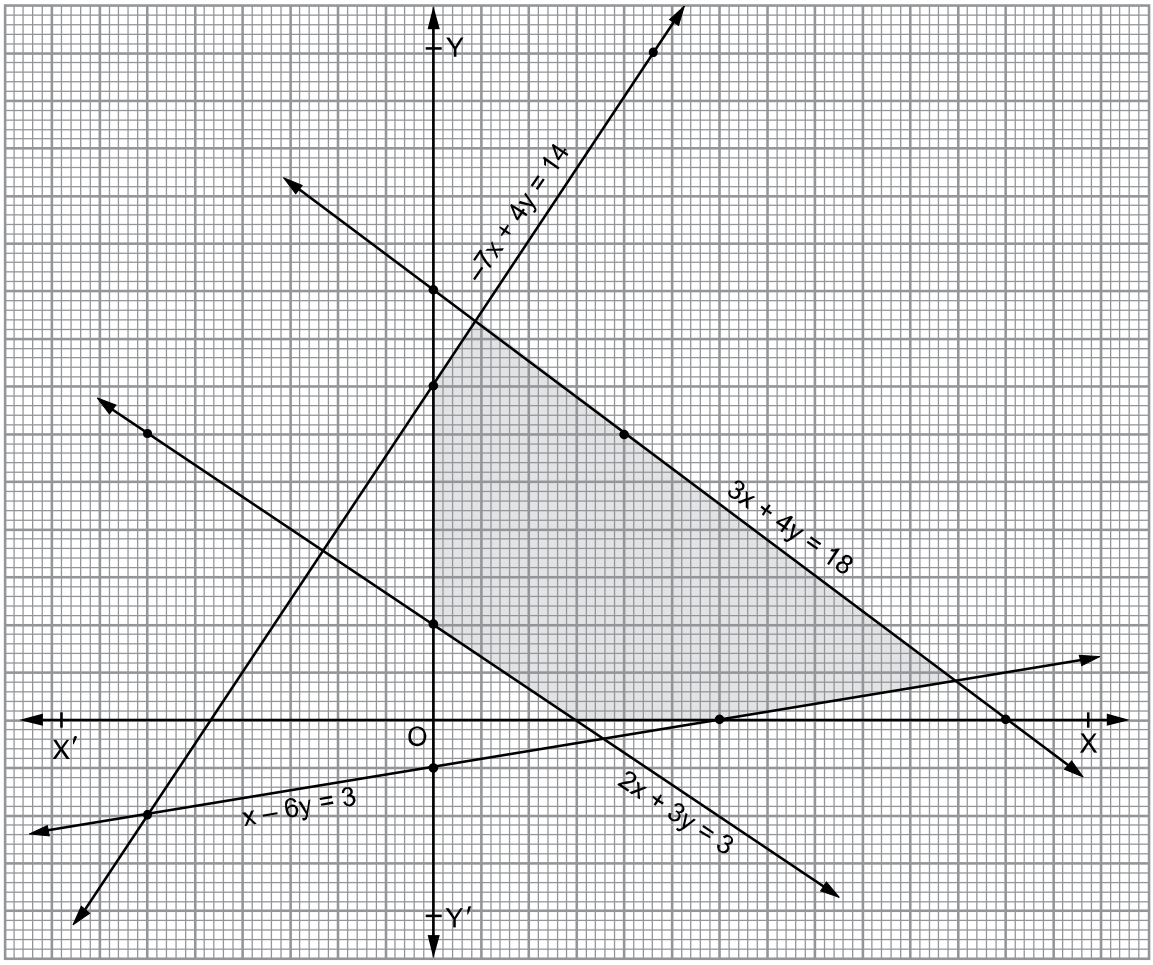
(iii) $x \geq 0$ is represented by the y-axis and the plane to its right.

(iv) $y \geq 0$ is represented by the x-axis and the plane above the x-axis.



It is clear from the above graph that the solution set of the given inequations is an unbounded region, shown by the shaded region.

EXAMPLE 12 *Find the linear inequations for which the shaded region in the figure given below, is the solution set.*



SOLUTION (i) Consider the line, $3x + 4y = 18$.

We observe here from the graph that the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

But, (0, 0) satisfies the inequation $3x + 4y \leq 18$.

So, the inequation corresponding to this line is $3x + 4y \leq 18$.

(ii) Consider the line, $2x + 3y = 3$.

We observe here from the graph that the shaded region and the origin $O(0, 0)$ lie on the opposite sides of this line.

Here, (0, 0) satisfies the inequation $2x + 3y \leq 3$.

So, the inequation corresponding to this line is $2x + 3y \geq 3$.

(iii) Consider the line, $-7x + 4y = 14$.

We observe here from the graph that the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

Here, (0, 0) satisfies the inequation $-7x + 4y \leq 14$.

So, the inequation corresponding to this line is $-7x + 4y \leq 14$.

(iv) Consider the line, $x - 6y = 3$.

We observe here from the graph that the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

Also, the point (0, 0) satisfies the inequation $x - 6y \leq 3$.

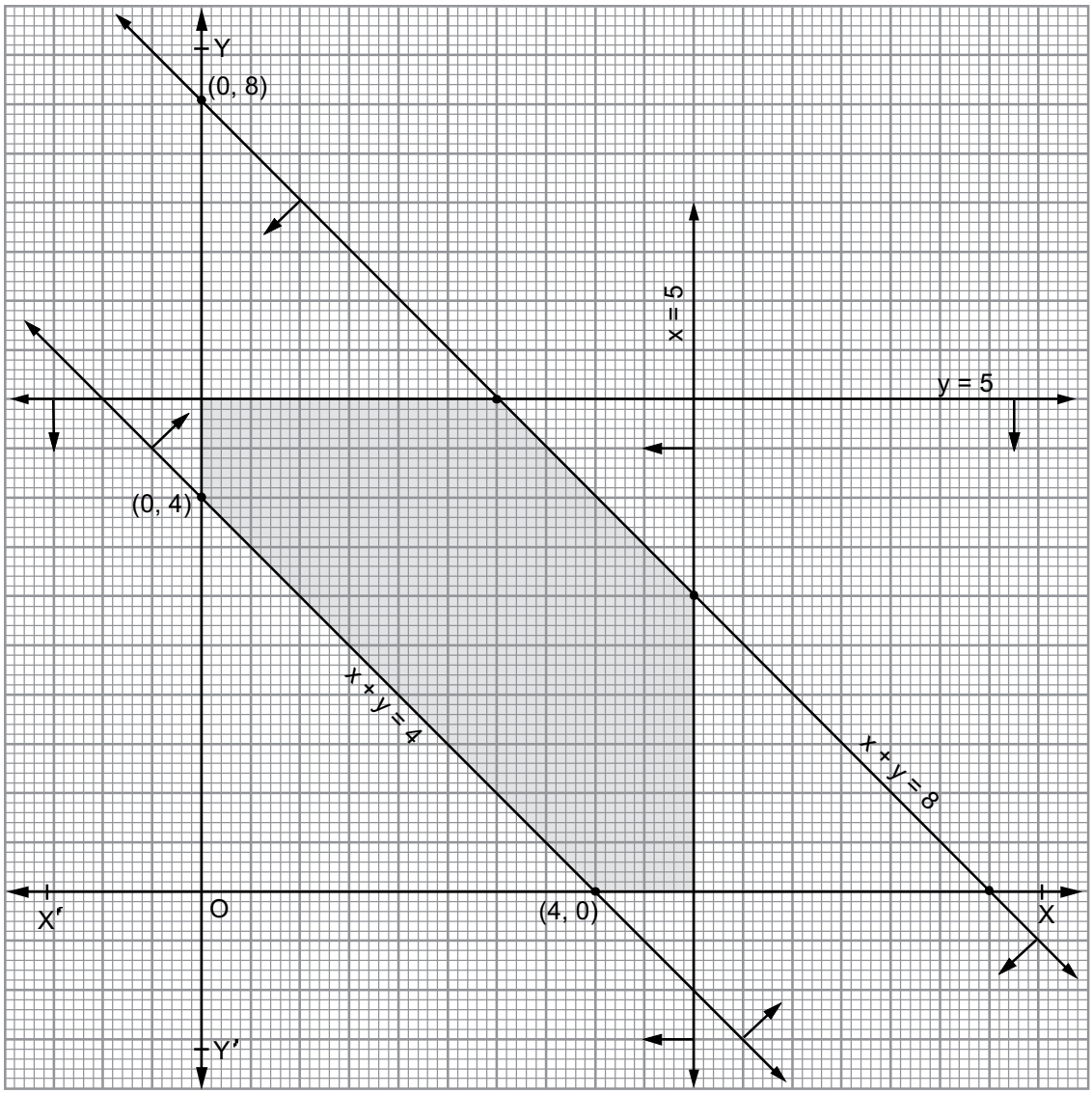
So, the inequation corresponding to this line is $x - 6y \leq 3$.

(v) It is clear from the given figure that the whole shaded region lies in the first quadrant only. So, $x \geq 0$ and $y \geq 0$.

Hence, the linear inequations corresponding to the given solution set are

$3x + 4y \leq 18$, $2x + 3y \geq 3$, $-7x + 4y \leq 14$, $x - 6y \leq 3$, $x \geq 0$ and $y \geq 0$.

EXAMPLE 13 *Find the linear inequations for which the shaded region of the figure given below, is the solution set.*



SOLUTION (i) Consider the graph line, $x = 5$.

It is clear that the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

And, the point (0, 0) satisfies the inequation $x \leq 5$.

So, the inequation corresponding to this line is $x \leq 5$.

(ii) Consider the graph line, $y = 5$.

Clearly, the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

And, the point (0, 0) satisfies the inequation $y \leq 5$.

So, the inequation corresponding to this line is $y \leq 5$.

(iii) Consider the graph line, $x + y = 8$.

Clearly, the shaded region and the origin $O(0, 0)$ lie on the same side of this line.

And, the point (0, 0) satisfies the inequation $x + y \leq 8$.

So, the inequation corresponding to this line is $x + y \leq 8$.

(iv) Consider the graph line, $x + y = 4$.

It is clear from the graph that the shaded region and the origin $O(0, 0)$ lie on the opposite sides of this line. Also, the point (0, 0) satisfies the inequation $x + y \leq 4$.

So, the inequation corresponding to this line is $x + y \geq 4$.

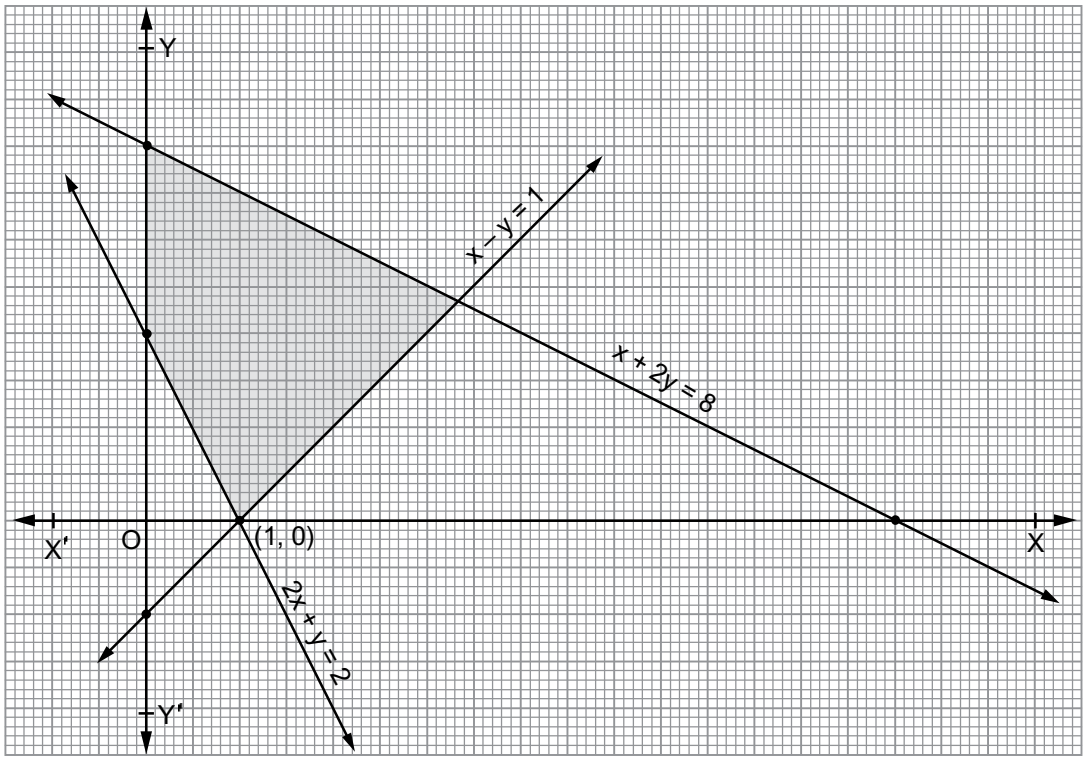
(v) It is clear from the given figure that the whole shaded region lies in the first quadrant only.

So, $x \geq 0$ and $y \geq 0$.

Hence, the linear inequations corresponding to the given solution set are

$$x \leq 5,\ y \leq 5,\ x + y \leq 8,\ x + y \geq 4,\ x \geq 0 \text{ and } y \geq 0.$$

EXAMPLE 14 *Find the linear inequations for which the shaded region of the figure given below, is the solution set.*



SOLUTION (i) Consider the graph line, $x + 2y = 8$.

It is clear that the shaded region and the origin $O(0, 0)$ lie on the same side of the line $x + 2y = 8$ and (0, 0) satisfies the inequation $x + 2y \leq 8$.

So, the inequation corresponding to this line is $x + 2y \leq 8$.

(ii) Consider the graph line, $2x + y = 2$.

It is clear that the shaded region and the origin $O(0, 0)$ lie on the opposite sides of the line $2x + y = 2$.

And, (0, 0) does not satisfy the inequation $2x + y \geq 2$.

So, the inequation corresponding to this line is $2x + y \geq 2$.

(iii) Consider the graph line, $x - y = 1$.

It is clear that the shaded region and the origin $O(0, 0)$ lie on the same side of the line $x - y = 1$.

And, (0, 0) satisfies the inequation $x - y \leq 1$.

So, the inequation corresponding to this line is $x - y \leq 1$.

(iv) It may be noted here that the shaded region is above the x-axis and is on the right side of the y-axis.

So, we must have $x \geq 0$ and $y \geq 0$.

Hence, the linear inequations corresponding to the given solution set are

$$x + 2y \leq 8,\ 2x + y \geq 2,\ x - y \leq 1,\ x \geq 0,\ y \geq 0.$$

EXAMPLE 15 *A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry and then sent to another machine for finishing. The number of man-hours of labour required in each shop for the production of each unit of A and B and the number of man-hours for the firm available per week are as follows:*

	Foundry	*Machine shop*
Man-hours for 1 unit of gadget A	10	5
Man-hours for 1 unit of gadget B	6	4
Firm's capacity per week in man-hours	1000	600

Formulate it in the form of linear inequations. Draw the graph showing the solution of all these inequations.

SOLUTION Suppose that the firm manufactures x units of A and y units of B.

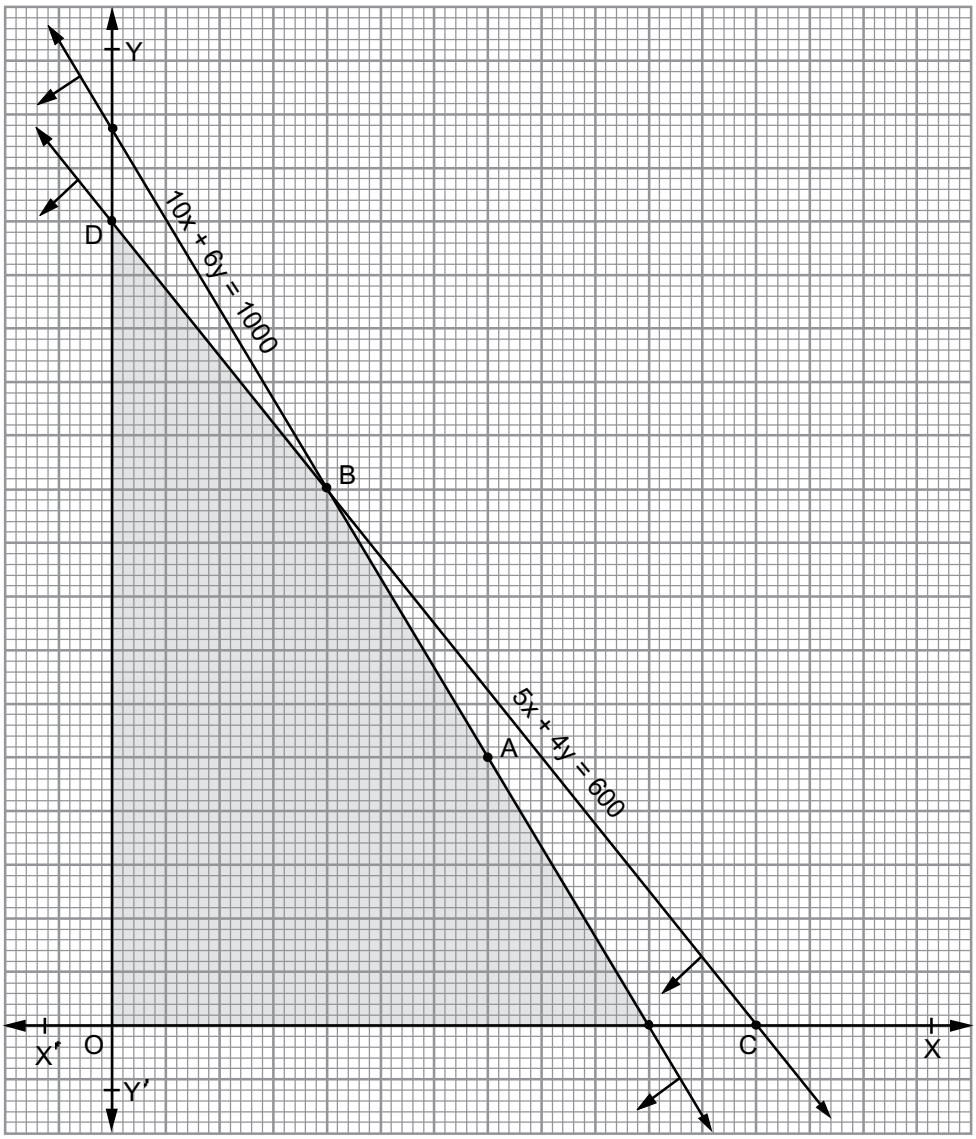
$\therefore$ number of man-hours for A and B in the foundry $= 10x + 6y$,

and number of man-hours for A and B in the machine shop $= 5x + 4y$.

$\therefore$ $10x + 6y \leq 1000$ and $5x + 4y \leq 600$.

Clearly, $x \geq 0$ and $y \geq 0$ [$\because$ number of units cannot be negative].

SCALE 1 small division = 2 units.



(i) First we draw the graph of $10x + 6y \leq 1000$.

Consider the equation, $10x + 6y = 1000$.

$10x + 6y = 1000 \Rightarrow 5x + 3y = 500$.

The values of (x, y) satisfying $5x + 3y = 500$ are:

x	70	40
y	50	100

Plot the points $A(70, 50)$ and $B(40, 100)$ on a graph paper and join them by the thick line AB.

Consider (0, 0). It does not lie on $5x + 3y = 500$.

Also, (0, 0) satisfies $10x + 6y \leq 1000$.

Therefore, the line AB and the part of the plane separated by AB, containing (0, 0), represent the graph of $10x + 6y \leq 1000$.

(ii) Now, we draw the graph of $5x + 4y \leq 600$.

Consider the line, $5x + 4y = 600$.

$$5x + 4y = 600 \Rightarrow \frac{x}{120} + \frac{y}{150} = 1.$$

This line meets the axes at $C(120, 0)$ and $D(0, 150)$.

Plot these points on the same graph paper as above and join them by the thick line CD.

Now, consider (0, 0). It does not lie on $5x + 4y = 600$.

Also, (0, 0) satisfies the inequation $5x + 4y \leq 600$.

So, the line CD and that part of the plane separated by CD, containing (0, 0), represent the graph of $5x + 4y \leq 600$.

$x \geq 0$ is represented by the y-axis and the part of the plane on its right side.

$y \geq 0$ is represented by the x-axis and the plane above the x-axis.

Clearly, the shaded region together with its boundary represents the solution set of the given inequations.

EXERCISE 7

Draw the graph of the solution set of each of the following inequations:

1. $x + y \geq 4$ **2.** $x - y \leq 3$ **3.** $y - 2 \leq 3x$

4. $x \geq y - 2$ **5.** $3x + 2y > 6$ **6.** $3x + 5y < 15$

Solve the following systems of inequations graphically:

7. $x \geq 2, y \geq 3$ **8.** $3x + 2y \leq 12, x \geq 1, y \geq 2$

9. $x + y \leq 6, x + y \geq 4$ **10.** $2x + y \geq 6, 3x + 4y \leq 12$

11. $x + y \leq 9, y > x, x \geq 0$ **12.** $2x - y > 1, x - 2y < -1$

13. $5x + 4y \leq 20, x \geq 1, y \geq 2$ **14.** $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$

15. $2x + y \geq 4, x + y \leq 3, 2x - 3y \leq 6$

16. $x + 2y \leq 10, x + y \geq 1, x - y \leq 0, x \geq 0, y \geq 0$

17. $4x + 3y \leq 60, y \geq 2x, x \geq 3, x \geq 0, y \geq 0$

18. $x - 2y \leq 2, x + y \geq 3, -2x + y \leq 4, x \geq 0, y \geq 0$

19. $x + 2y \leq 100, 2x + y \leq 120, x + y \leq 70, x \geq 0, y \geq 0$

20. $x + 2y \leq 2000, x + y \leq 1500, y \leq 600, x \geq 0, y \geq 0$

21. Show that each of the following systems of linear inequations has no solution:

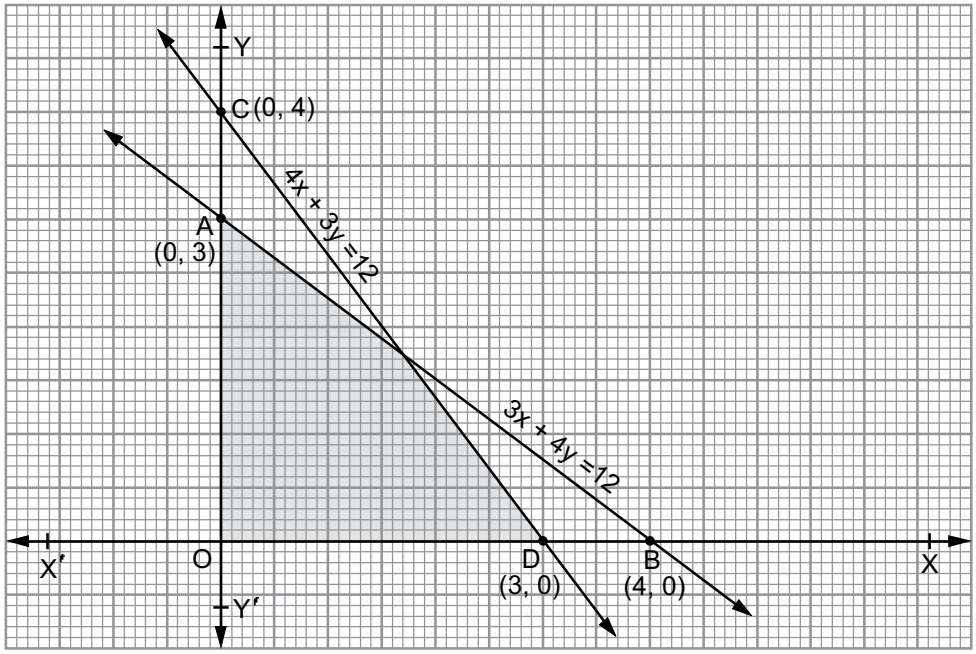
(i) $3x + 2y \geq 24, 3x + y \leq 15, x \geq 4$

(ii) $2x - y \leq -2, x - 2y \geq 0, x \geq 0, y \geq 0$

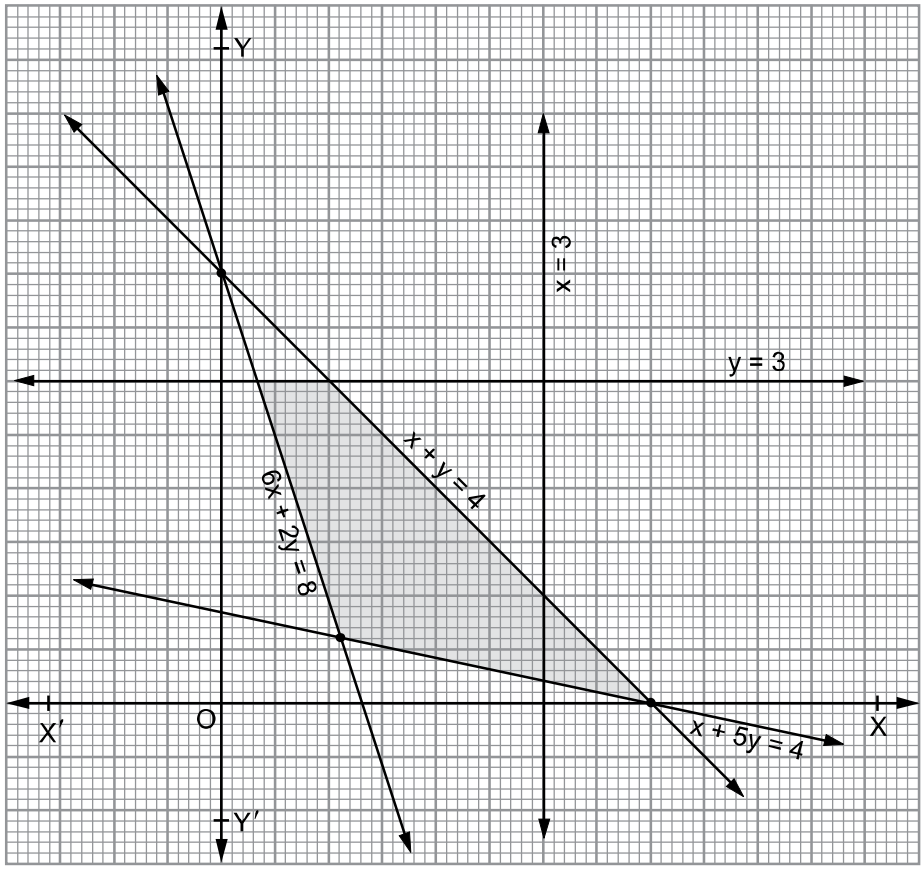
22. Show that the solution set of the following system of inequations is an unbounded set:

$3x + y \geq 12, x + y \geq 9, x \geq 0, y \geq 0.$

23. Find the linear inequations for which the shaded area is the solution set in the figure given below.



24. Find the linear inequations for which the shaded area in the figure given below, is the solution set.



25. A furniture dealer deals in only two items: tables and chairs. He has ` 30000 to invest and a space to store at most 60 pieces. A table costs him ` 1500 and a chair ` 300. Formulate the data in the form of inequations and draw a graph representing the solution of these inequations.

Hint Suppose he makes x tables and y chairs. Then,

$$1500x + 300y \leq 30000,\ x + y \leq 60,\ x \geq 0 \text{ and } y \geq 0.$$

26. If a young man rides his motorcycle at 40 km per hour, he has to spend ` 6 per km on petrol and if he rides it at 50 km per hour, the petrol cost rises to ` 10 per km. He has ` 500 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Formulate the data in the form of inequations and draw a graph representing the solution of these inequations.

Hint Suppose he rides x km at 40 km/ hr and y km at 50 km/ hr.

Then, $6x + 10y \leq 500,\ \frac{x}{40} + \frac{y}{50} \leq 1,\ x \geq 0,\ y \geq 0.$

ANSWERS (EXERCISE 7)

23. $3x + 4y \leq 12,\ 4x + 3y \leq 12,\ x \geq 0,\ y \geq 0$

24. $x + y \leq 4,\ x + 5y \geq 4,\ 6x + 2y \geq 8,\ x \leq 3,\ y \leq 3,\ x \geq 0,\ y \geq 0$